Universidad Autónoma de Querétaro

Facultad de Ingeniería

Licenciatura en Ingeniería Civil

"DESIGN OF CONCRETE STRUCTURES"

GUIA DEL MAESTRO

QUE PARA OBTENER EL GRADO DE

INGENIERO CIVIL

Presenta:

Francisco Javier Meneses García

DIRIGIDA POR:

Dr. Enrique Rico García

C.U. QUERÉTARO, QUERÉTARO. 2012

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BASED ON RFD 2004

PREFACE

"Assisted learning or guided participation in the class room requires instructional scaffolding, in other words give information, indicators, reminders and motivation at the exact moment. Then gradually allow students to make homework and assignments by their own" (Anita E Woolfolk, 2006). "The teachers promote learning by adapting the material or the exercises at the current level that student has. To demonstrate skills and mental processes; taking the students through the steps a complicated problem; solving part of the problem; giving accurate feedback and allowing review, or proposing questions that change the point of view of the student" (Rosenshine y Meiter, 1992).

This is the main objective by making this teachers guide, which gives the necessary information and helps the students step by step to solve a problem, also the student can get feedback of what they saw in class, creating an enhanced learning

The teacher's guide of concrete structures proposed a solid base for the student and for the professor to be taught in class.

So the objectives that we have in this guide are:

Have a guide in which the students and the professor can obtain information about the subject, it would also serve as a glossary in which as a student can know the correct translation of the words related to the subject and/or the construction area, because the imparted English is conventional and no technical, which is necessary for the student development in this area.

A guide in which the student can find important information necessary for the concrete structures subject

Take the student step by step in the exercises. Explain the procedure of the problems with the objective that the students get a better comprehension of the problem.

Chapter I

Introduction

I. INTRODUCTION

Concrete

Concrete is a stone like material obtained by permitting a carefully proportioned mixture of cement, sand and gravel or other aggregates, and water to harden in forms of the shape and dimensions of the desired structure.

The bulk of the material consists of fine and coarse aggregate. Cement and water interact chemically to bind the aggregate particles into a solid mass. Additional water, over and above that needed for this chemical reaction, is necessary to give the mixture the workability that enables it to fill the forms and surround the embedded reinforcing steel prior to hardening.

Concretes with a wide range of properties can be obtained by appropriate adjustment of the proportions of the constituents materials. Special cements (such as high early strength cements), special aggregates (such as plasticizers, air-entraining agents, silica fume, and fly ash), and special curing methods (such as steam-curing) permit a wide variety of properties to be obtained.

These properties depend to a very substantial degree:

- On the proportions of the mix
- On the care with the various constituents are intermix
- On the conditions of humidity and temperature in which the mix is maintained from the moment it is placed in the forms until the completions of curing.

The process of controlling conditions after placement is known as curing.

To protect against the unintentional production of substandard concrete, a high degree of skillful control and supervision is necessary throughout the process, from the proportioning of the individual materials, through mixing and placing, until the completion of curing

The factors that make concrete a universal building material are so pronounced:

- The facility with which, while plastic, it can be deposited and made to fill forms or molds of any practical shape.
- Its high fire and weathering resistance.
- Most of the constituent materials, with the exception of cement and additives, are usually available at low cost locally or at small distances from the construction site.
- Its compressive strength, like that of natural stones, which makes it suitable for members primarily subject to compression, such as columns and arches. On the other hand, again as in natural stones, it is a relatively brittle material whose tensile strength is small compared with its compressive strength.

Reinforced concrete

To offset this limitation steel is used, with its high tensile strength, to reinforce concrete, in those places where its low tensile strength would limit the carrying capacity of the member.

The steel, usually round rods with appropriate surface deformation to provide interlocking, is placed in the forms in advance of the concrete. When completely surrounded by the harden concrete mass, it forms an integral part of the member.

The resulting combination of these two materials, known as reinforced concrete, combines many of the advantages of each:

Concrete

- Low cost
- Good weather and fire resistance
- Good compressive strength
- Excellent formability

Steel

- High tensile strength
- Great ductility
- Toughness

It is this combination that allows the almost unlimited range of uses and possibilities of reinforced concrete in the construction of buildings, bridges, dams, tanks, reservoirs, and a host of other structures.

Loads

Loads that act on structures can be divided into three categories: dead loads, live loads, and environmental loads.

Dead loads

Are those that are constant in magnitude and fixed in location throughout the lifetime of the structure. Usually the major part of the dead load is the weight of the structure itself. This can be calculated with good accuracy from the design configuration, dimensions of the structure, and density of the material. For buildings, floor fill, finish floors, and plastered ceilings are usually included as dead loads, and allowance is made for suspended loads, such as piping and lighting fixtures.

Material Designation	Specific Gravity Kg/ Cm3
Cement and Sand	2100
Cement, Lime and Sand	1900
Lime and Sand	1700
Lime , Sand and Brick Dust	1600

Table SG

Live loads

Consist chiefly of occupancy loads in buildings. They may be either fully or partially in place or not present at all, and may also change in location. Their magnitude and distribution at any given time are uncertain, and even their maximum intensities throughout the lifetime of the structure are not known with precision. These loads are specified in the building codes that governs at the site of construction.

Tabulated live loads cannot always be used. The type of occupancy should be considered and the probable loads computed as accurately as possible. Warehouses for heavy storage may be design for loads as high as 24000 kg m^{-2} so special provisions must be made for all definitely located heavy concentrated loads.

TABLE LIVE LOADS (Kg/m2)

Destino de piso o cubierta	W	W_a	W_m	Obser-vacio-nes
a) Habitación (casa–habitación, departa-mentos, viviendas, dormitorios, cuartos de hotel, internados de escuelas, cuarteles, cárceles, correccionales, hospitales y similares)	0.7 (70)	0.9 (90)	1.7 (170)	1
b) Oficinas, despachos y laboratorios	1.0 (100)	1.8 (180)	2.5 (250)	2
c) Aulas	1.0 (100)	1.8 (180)	2.5 (250)	
d) Comunicación para	0.4	1.5	3.5	3 y 4

	peatones (pasillos, escaleras, rampas, vestíbulos y pasajes de acceso libre al público)	(40)	(150)	(350)	
e)	Estadios y lugares de reunión sin asientos individuales	0.4 (40)	3.5 (350)	4.5 (450)	5
f)	Otros lugares de reunión (bibliotecas, templos, cines, teatros, gimnasios, salones de baile, restaurantes, salas de juego y similares)	0.4 (40)	2.5 (250)	3.5 (350)	5
g)	Comercios, fábricas y bodegas	$0.8W_m$	$0.9W_m$	W_m	6
h)	Azoteas con pendiente no mayor de 5 %	0.15 (15)	0.7 (70)	1.0 (100)	4 y 7
i)	Azoteas con pendiente mayor de 5 %; otras cubiertas, cualquier pendiente.	0.05 (5)	0.2 (20)	0.4 (40)	4, 7, 8 y 9
j)	Volados en vía pública (marquesinas, balcones y similares)	0.15 (15)	0.7 (70)	3 (300)	
k)	Garajes y estacionamientos (exclusivamente para automóviles)	0.4 (40)	1.0 (100)	2.5 (250)	10

Table CVU from RCDF

Environmental loads

Consist mainly of snow loads, wind pressure and suction, earthquake loads, soil pressures on surface portions of a structure, loads for possible ponding of rain water on flat surfaces, and forces caused by temperature differentials. Like live loads, environmental loads at any given time are uncertain both in magnitude and distribution.

Serviceability, strength, and structural safety

To serve its purpose, a structure must be safe against collapse and serviceable in use. Serviceability requires that deflections be adequately small; that cracks, if any, be kept to tolerable limits, that vibrations be minimized; etc.

Safety requires that the strength of the structure be adequate for all loads that may foreseeably act on it.

If the strength of the structure, built as designed, could be predicted accurately, and if the loads and their internal effects (moments, shears, and axial forces) were known accurately, safety could be ensured by providing a carrying capacity just barely in excess of the known loads. However, there are a number of sources of uncertainty in the analysis, design, and construction of reinforced concrete structures. These sources of uncertainty, which require a definite margin of safety, may be listed as follows:

- Actual loads may differ from those assumed.
- Actual loads may be distributed in a manner different from that assumed.
- The assumptions and simplification inherent in analysis may result in calculated loads effects (moments, shears and axial forces) different from those that, in fact, act in the structure.
- The actual structural behavior may differ from that assumed, owing to imperfect knowledge.
- Actual member dimensions may differ from those specified
- Reinforcement may not be in its proper position.
- Actual material strength may be different from that specified.

It is evident that the selection of an appropriate margin of safety is not a simple matter.

From the aforementioned points it is clear that loads acting on a structure and the strength of a structure are both random variables.

In spite of this uncertainty, the engineer must provide an adequate structure.

Structural safety

A given structure has a safety margin M if

$$M = S - Q > 0$$

That is, if the strength of the structure is larger than the load acting on it. Since S and Q are random variables M is too a random variable.

Failure occurs when M is less than zero. The random nature of the problem needs to be taken into account by using safety coefficients

$$\varphi S \geq \gamma Q$$

Where φ is a partial safety coefficient smaller than one applied to the strength S, and γ is a partial safety coefficient larger than one applied to the load Q.

Furthermore, recognizing the differences in variability between, say dead loads D, lived loads L and environmental loads W, it is both reasonable and easy to introduce different loads factors for different types of loads.

$$\varphi S \geq \gamma_1 D + \gamma_2 L + \gamma_3 W$$

Now the reduce probability that maximum dead, live and wind or other loads will act simultaneously can be incorporated by including a factor α less than one

$$\varphi S \geq \alpha\,(\gamma_1 D + \gamma_2 L + \gamma_3 W)$$

Fundamentals assumptions for reinforced concrete behavior

Design is the determination of the general shape and all specific dimensions of a particular structure so that it will perform the function for which it is created and will safely withstand the influences that will act on it throughout its useful life. The chief items of behavior that are of practical interest are:

- The strength of the structure (safety)
- Deformation that the structure will undergo when loaded under service conditions (serviceability)

The fundamentals propositions on which the mechanics of reinforced concrete is based are as follows:

- The internal forces, such as bending moments, shear forces, and normal and shear stresses, at any section of a member are in equilibrium with the effects of the external loads at that section. This proposition is not an assumption but a fact.

- The strain in an embedded reinforcing steel bar (unit extension or compression) is the same as that of the surrounding concrete. In other words, it is assumed that perfect bonding exists between concrete and steel at the interface, so that no slip can occur between the two materials.

- Cross sections that were plane prior to loading continue to be plane in the member under load.

- It is assumed that concrete is not capable to resist any tension stress whatever. This assumption is evidently a simplification.

Actually, the joint of action of two materials as dissimilar and complicated as concrete and steel is so complex that it has not yet lent itself to purely analytical treatment. For this reason methods of design and analysis, while using these assumptions, are very largely based on the results of extensive and continuing experimental research. They are modified and improved as additional test evidence becomes available.

Concrete materials

A) Cementitious

A cementations material is one that has the adhesive and cohesive properties necessary to bond inert aggregates into a solid mass of adequate strength and durability.

For making structural concrete, so-called hydraulic cements are used exclusively. Water is needed for the chemical process (hydration) in which the cement powder sets and hardens into one solid mass.

Of the various hydraulic cements that have been developed, Portland cement is the most common. Portland cement is a finely powdered, grayish material that consists chiefly of calcium and aluminum silicates. The common raw materials from which it is made are limestones, and clays.

When cement is mixed with water to form a soft paste, it gradually stiffens until it becomes a solid. This process is known as setting and hardening. The cement is said to have set when it has gained sufficient rigidity to support an arbitrarily defined pressure, after which it continues for a long time to harden, that is, to gain further strength.

The water in the paste dissolves material at the surfaces of the cement grains and forms a gel that gradually increases in volume and stiffness. This leads to a rapid stiffening of the paste 2 to 4 hours after water has been added to the cement.

Hydration continues to proceed deeper into the cement grains, at decreasing speed, with continued stiffing and hardening of the mass.

For complete hydration of a given amount of cement, an amount of water equal to about 25 percent of that of cement, by weight is needed chemically. An additional amount must be present, however to provided mobility for the water in the cement paste during the hydration process so that it can reach the cement particles and to provide the necessary workability of the concrete.

For normal concretes, the water-cement ratio is generally in the range of about 0.40 to 0.60, although for high-strength concretes, ratios as low as 0.21 have been used. In this case, the needed workability is obtained through the use of admixtures.

Any amount of water above that consumed in the chemical reaction produces pores in the cement paste. The strength of the hardened paste decreases in inverse proportion to the fraction of the total volume occupied by pores. That is why the strength of the cement paste depends primarily on, and decreases directly with, an increasing water-cement ratio. See Figure 1.1.

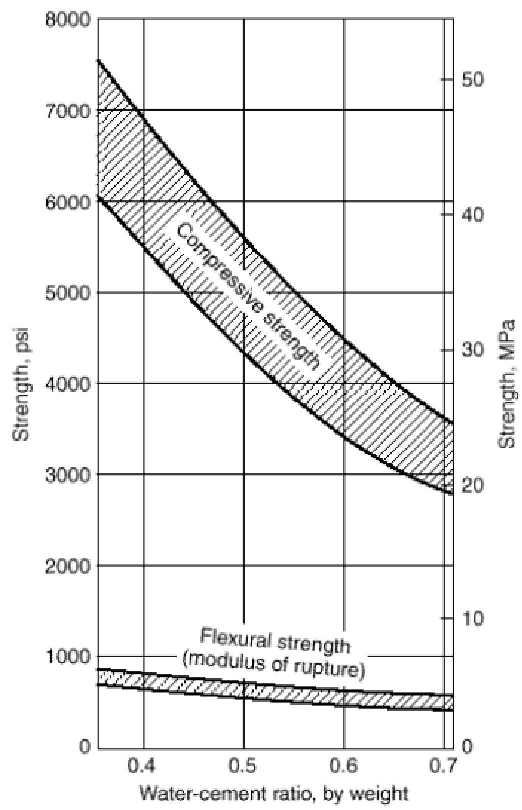


Figure 1.1 Effect of water-cement ratio on 28-day compressive and flexural tensile strength.

B) Aggregates

In ordinary structural concretes the aggregates occupy about 70 to 75 percent of the volume of the harden mass. The remainder consist of hardened cement paste, uncombined water (that is, water not involved in the hydration process) and air voids. The latter two evidently do not contribute to the strength of the concrete. In general, the more densely the aggregate can be packed, the better the durability and economy of the concrete.

For this reason gradation of the particle sizes in the aggregate, to produce close packing, is of considerable important.

It is also important that the aggregate has good strength, durability, and weather resistant; that its surfaces are free of impurities that may weaken the bond with cement paste.

Natural aggregates are generally classified as fine and coarse. Fine aggregate (typically natural sand) is any material that will pass a No. 4 sieve (four openings per linear inch). Material coarser than that is classified as coarse aggregate.

The maximum size of coarse aggregate in reinforced concrete is governed by the requirement that it shall easily fit into the forms and between the reinforcing bars. For this purpose it should not be larger than one-fifth of the narrowest dimension of the forms or one third of the deep of slabs, nor three-quarters of the minimum distance between reinforcing bars.

Proportioning of Concrete

Scope

This Standard Practice describes methods for selecting proportions for hydraulic cement concrete made with and without other cementitious materials and chemical admixtures.

The methods provide a first approximation of proportions intended to be checked by trial batches in the laboratory or field and adjusted, as necessary, to produce the desired characteristics of the concrete.

Introduction

Concrete is composed principally of aggregates, a portland or blended cement, and water, and may contain other cementitious materials and/or chemical admixtures. It will contain some amount of entrapped air and may also contain purposely entrained air obtained by use of an admixture or air-entraining cement. Chemical admixtures are frequently used to accelerate, retard, improve workability, reduce mixing water requirements, increase strength, or alter other properties of the concrete.

The selection of concrete proportions involves a balance between economy and requirements for placeability, strength, durability, density, and appearance. The required characteristics are governed by the use to which the concrete will be put and by conditions expected to be encountered at the time of placement. These characteristics should be listed in the job specifications.

The ability to tailor concrete properties to job needs reflects technological developments that have taken place, for the most part, since the early 1900s. The use of water cement ratio as a tool for estimating strength was recognized about 1918. The remarkable improvement in durability resulting from the entrainment of air was recognized in the early 1940s. These two significant developments in concrete technology have been augmented by extensive research and development in many related areas, including the use of admixtures to counteract possible deficiencies, develop special properties, or achieve economy.

Proportions calculated by any method must always be considered subject to revision on the basis of experience with trial batches.

<u>Basic relationships</u>

Concrete proportions must be selected to provide necessary placeability, density, strength, and durability for the particular application.

- Placeability (including satisfactory finishing properties) encompasses traits loosely accumulated in the terms “workability” and “consistency.” For the purpose of this discussion, workability is considered to be that property of concrete that determines its capacity to be placed and consolidated properly and to be finished without harmful segregation. It embodies such concepts as moldability, cohesiveness, and compactability. Workability is affected by: the grading, particle shape, and proportions of aggregate; the amount and qualities of cement and other cementitious materials; the presence of entrained air and chemical admixtures; and the consistency of the mixture.

- Consistency -- Loosely defined, consistency is the relative mobility of the concrete mixture. It is measured in terms of slump -- the higher the slump the more mobile the mixture -- and it affects the ease with which the concrete will flow during placement. It is related to but not synonymous with workability. In properly proportioned concrete, the unit water content required to produce a given slump will depend on several factors. Water requirement increases as aggregates become more angular and rough textured (but this disadvantage may be offset by improvements in other characteristics such as bond to cement paste). Required mixing water decreases as the maximum size of well-graded aggregate is increased. It also decreases with the entrainment of air. Mixing water requirements usually are reduced significantly by certain chemical water-reducing admixtures.

- Strength -- Strength at the age of 28 days is frequently used as a parameter for the structural design, concrete proportioning, and evaluation of concrete. These may be related to strength in a general way, but are also affected by factors not significantly associated with strength.

In mass concrete, mixtures are generally proportioned to provide the design strength at an age greater than 28 days. However, proportioning of mass concrete should also provide for adequate early strength as may be necessary for form removal and form anchorage.

- Water-cement or water-cementitious ratio [w/c or w/(c + p)] -- For a given set of materials and conditions, concrete strength is determined by the net quantity of water used per unit quantity of cement or total cementitious materials. The net water content excludes water absorbed by the aggregates. Differences in strength for a given water cement ratio w/c or water-cementitious materials ratio w/(c + p) may result from changes in: maximum size of aggregate; grading, surface texture, shape, strength, and stiffness of aggregate particles; differences in cement types and sources; air content; and the use of chemical admixtures that affect the cement hydration process or develop cementitious properties themselves.

- Durability -- Concrete must be able to endure those exposures that may deprive it of its serviceability – freezing and thawing, wetting and drying, heating and cooling, chemicals, deicing agents, and the like.

Use of low water-cement or cementitious materials ratio [w/c or w/(c + p)] will prolong the life of concrete by reducing the penetration of aggressive liquids. Resistance to severe weathering, particularly freezing and thawing, and to salts used for ice removal is greatly improved by incorporation of a proper distribution of entrained air. Entrained air should be used in all exposed concrete in climates where freezing occurs.

- Density -- For certain applications, concrete may be used primarily for its weight characteristic. Examples of applications are counterweights on lift bridges, weights for sinking oil pipelines under water, shielding from radiation, and insulation from sound.

- Generation of heat -- A major concern in proportioning mass concrete is the size and shape of the completed structure or portion thereof. Concrete placements large enough to require that measures be taken to control the generation of heat and resultant volume change within the mass will require consideration of temperature control measures.

Effect of chemical admixtures, pozzolanic, and other materials on concrete proportions

- Admixtures -- By definition (ACI 116R), an admixture is "a material other than water, aggregates, hydraulic cement, and fiber reinforcement used as an ingredient of concrete or mortar and added to the batch immediately before or during its mixing." Consequently, the term embraces an extremely broad field of materials and products, some of which are widely used while others have limited application.

- Air-entraining admixture -- Air-entrained concrete is almost always achieved through the use of an air-entraining admixture. The use of an air-entraining admixture gives the concrete producer the flexibility to adjust the entrained air content to compensate for the many conditions affecting the amount of air entrained in concrete, such as: characteristics of aggregates, nature and proportions of constituents of the concrete admixtures, type and duration of mixing, consistency, temperature, cement fineness and chemistry, use of other cementitious materials or chemical admixtures, etc.

Because of the lubrication effect of the entrained air bubbles on the mixture and because of the size and grading of the air voids, air-entrained concrete usually contains up to 10 percent less water than non-air-entrained concrete of equal slump. This reduction in the volume of mixing water as well as the volume of entrained and entrapped air must be considered in proportioning.

- Chemical admixtures -- Since strength and other important concrete qualities such as durability, shrinkage, and cracking are related to the total water content and the w/c or w/(c + p), water-reducing admixtures are often used to improve concrete quality. Further, since less cement can be used with reduced water content to achieve the same w/c or w/(c + p) or strength, water-reducing and set-controlling admixtures are used widely for reasons of economy. Chemical admixtures types A through G, are of many formulations and their purposes for use in concrete are as follows:

Type A -- Water-reducing

Type B -- Retarding

Type C – Accelerating

Type D -- Water-reducing and retarding

Type E -- Water-reducing, and accelerating

Type F -- Water-reducing, high-range

Type G -- Water-reducing, high-range, and retarding

Background data

To the extent possible, selection of concrete proportions should be based on test data or experience with the materials actually to be used. Where such background is limited or not available, estimates given in this recommended practice may be employed.

The following information for available materials will be useful:

- Sieve analyses of fine and coarse aggregates.
- Unit weight of coarse aggregate.
- Bulk specific gravities and absorptions of aggregates.
- Mixing-water requirements of concrete developed from experience with available aggregates.
- Relationships between strength and water-cement ratio or ratio of water-to-cement plus other cementitious materials, for available combinations of cements, other cementitious materials if considered, and aggregates.
- Specific gravities of Portland cement and other cementitious materials, if used.

Optimum combination of coarse aggregates to meet the maximum density gradings for mass concrete

Procedure

The procedure for selection of mix proportions given in this section is applicable to normal weight concrete. Although the same basic data and procedures can be used in proportioning heavyweight and mass concretes.

Estimating the required batch weights for the concrete involves a sequence of logical, straightforward steps which, in effect, fit the characteristics of the available materials into a mixture suitable for the work.

Step.-1 Choice of slump -- If slump is not specified, a value appropriate for the work can be selected from Table 2.1. The slump ranges shown apply when vibration is used to consolidate the concrete. Mixes of the stiffest consistency that can be placed efficiently should be used.

Table 2.1 Recommended slumps for various types of construction (SI)

Types of construction	Slump, mm	
	Maximum*	**Minimum**
Reinforced foundation walls and footings	75	25
Plain footings, caissons, and substructure walls	75	25
Beams and reinforced walls	100	25
Building columns	100	25
Pavements and slabs	75	25
Mass concrete	75	25

*May be increased 25 mm for methods of consolidation other than vibration

Step .- 2 Choice of maximum size of aggregate -- Large nominal maximum sizes of well graded aggregates have less voids than smaller sizes. Hence, concretes with the larger-sized aggregates require less mortar per unit volume of concrete. Generally, the nominal maximum size of aggregate should be the largest that is economically available and consistent with dimensions of the structure.

In no event should the nominal maximum size exceed:

- One-fifth of the narrowest dimension between sides of forms.
- One-third the depth of slabs.
- Nor three-fourths of the minimum clear spacing between individual reinforcing bars, bundles of bars, or pretensioning strands.

When high strength concrete is desired, best results may be obtained with reduced nominal maximum sizes of aggregate since these produce higher strengths at a given water-cement ratio.

Step .-3 Estimation of mixing water and air content -- The quantity of water per unit volume of concrete required to produce a given slump is dependent on: the nominal maximum size, particle shape, and grading of the aggregates; the concrete temperature; the amount of entrained air; and use of chemical admixtures.

Table 2.2 provides estimates of required mixing water for concrete made with various maximum sizes of aggregate, with and without air entrainment. Depending on aggregate texture and shape, mixing water requirements may be somewhat above or below the tabulated values, but they are sufficiently accurate for the first estimate.

A rounded and an angular coarse aggregate, both well and similarly graded and of good quality, can be expected to produce concrete of about the same compressive strength for the same cement factor in spite of differences in w/c or w/(c + p) resulting from the different mixing water requirements.

Table 2.2 Approximate mixing water and air content requirements for different slumps and nominal maximum sizes of aggregates (SI)

	Water, Kg/m^3 of concrete for indicated nominal maximum sizes of aggregate							
Slump, mm	9.5*	12.5*	19*	25*	37.5*	50†*	75†‡	150†‡
	Non-air-entrained concrete							
25 to 50	207	199	190	179	166	154	130	113
75 to 100	228	216	205	193	181	169	145	124
150 to 175	243	228	216	202	190	178	160	—
Approximate amount of entrapped air in non-air-entrained concrete, percent	3	2.5	2	1.5	1	0.5	0.3	0.2
	Air-entrained concrete							
25 to 50	181	175	168	160	150	142	122	107
75 to 100	202	193	184	175	165	157	133	119
150 to 175	216	205	197	184	174	166	154	—
Recommended average§ total air content, percent for level of exposure:								
Mild exposure	4.5	4.0	3.5	3.0	2.5	2.0	1.5**††	1.0**††
Moderate exposure	6.0	5.5	5.0	4.5	4.5	4.0	3.5**††	3.0**††
Extreme exposure‡‡	7.5	7.0	6.0	6.0	5.5	5.0	4.5**††	4.0**††

Step .-4 Selection of water-cement or water cementitious materials ratio -- The required w/c or w/(c + p) is determined not only by strength requirements but also by factors such as durability. Since different aggregates, cements, and cementitious materials generally produce different strengths at the same w/c or w/(c + p), it is highly desirable to have or to develop the relationship between strength and w/c or w/(c + p) for the materials actually to be used. In the absence of such data, approximate and relatively conservative values for concrete containing Type I Portland cement can be taken from Table 2.3. With typical materials, the tabulated w/c or w/(c + p) should produce the strengths shown, based on 28-day tests of specimens cured under standard laboratory conditions.

Table 2.3 Relationship between water-cement ratio and Compressive strength of concrete (SI)

	Water-cement ratio, by mass	
Compressive strength at 28 days, MPa*	Non-air-entrained concrete	Air-entrained concrete
40	0.42	—
35	0.47	0.39
30	0.54	0.45
25	0.61	0.52
20	0.69	0.60
15	0.79	0.70

For severe conditions of exposure, the w/c or w/(c + p) ratio should be kept low even though strength requirements may be met with a higher value. Table 2.4 gives limiting values.

*Table 2.4 Maximun permisible wáter-cement ratios for concrete in severe exposures (SI)**

Type of structure	Structure wet continuously or frequently and exposed to freezing and thawing†	Structure exposed to sea water or sulfates
Thin sections (railings, curbs, sills, ledges, ornamental work) and sections with less than 5 mm cover over stee	0.45	0.40‡
All other structures	0.50	0.45‡

***S**tep .-5 Calculation of cement content – The amount of cement per unit volume of concrete is fixed by the determinations made in Steps 3 and 4 above. The required cement is equal to the estimated mixing-water content (Step 3) divided by the water-cement ratio (Step 4).

If, however, the specification includes a separate minimum limit on cement in addition to requirements for strength and durability, the mixture must be based on whichever criterion leads to the larger amount of cement.

Step 6. Estimation of coarse aggregate content -- Aggregates of essentially the same nominal maximum size and grading will produce concrete of satisfactory workability when a given volume of coarse aggregate, on an oven-dry-rodded basis, is used per unit volume of concrete. Appropriate values for this aggregate volume are given in Table A1.5.3.6. It can be seen that, for equal workability, the volume of coarse aggregate in a unit volume of concrete is dependent only on its nominal maximum size and the fineness modulus of the fine aggregate.

The volume of aggregate in m3, on an oven-dry-rodded basis, for a m3 of concrete is equal to the value from Table 2.5 multiplied by the dry-rodded unit mass. This volume is converted to dry weight of coarse aggregate required in a m3 of concrete by multiplying it by the oven-dry-rodded weight per m3 of the coarse aggregate.

Table 2.5 Volume of coarse aggregate per unit of volume of concrete (SI)

Nominal maximum size of aggregate, mm	Volume of dry-rodded coarse aggregate* per unit volume of concrete for different fineness moduli† of fine aggregate			
	2.40	2.60	2.80	3.00
9.5	0.50	0.48	0.46	0.44
12.5	0.59	0.57	0.55	0.53
19	0.66	0.64	0.62	0.60
25	0.71	0.69	0.67	0.65
37.5	0.75	0.73	0.71	0.69
50	0.78	0.76	0.74	0.72
75	0.82	0.80	0.78	0.76
150	0.87	0.85	0.83	0.81

Step .-7 Estimation of fine aggregate content – At completion of Step 6, all ingredients of the concrete have been estimated except the fine aggregate. Its quantity is determined by difference. Either of two procedures may be employed: the weight method or the absolute volume method.

Weight Method

If the weight of the concrete per unit volume is assumed or can be estimated from experience, the required weight of fine aggregate is simply the difference between the weight of fresh concrete and the total weight of the other ingredients. Often the unit weight of concrete is known with reasonable accuracy from previous experience with the materials. In the absence of such information, Table 2.6 can be used to make a first estimate. Even if the estimate of concrete weight per m3 is rough, mixture proportions will be sufficiently accurate to permit easy adjustment on the basis of trial batches.

Table 2.6 First estimate of mass of fresh concrete (SI)

Nominal maximum size of aggregate, mm	First estimate of concrete unit mass, kg/m^{3}*	
	Non-air-entrained concrete	Air-entrained concrete
9.5	2280	2200
12.5	2310	2230
19	2345	2275
25	2380	2290
37.5	2410	2350
50	2445	2345
75	2490	2405
150	2530	2435

Absolute volume method

A more exact procedure for calculating the required amount of fine aggregate involves the use of volumes displaced by the ingredients. In this case, the total volume displaced by the known ingredients (water, air, cementitious materials, and coarse aggregate) is subtracted from the unit volume of concrete to obtain the required volume of fine aggregate. The volume occupied in concrete by any ingredient is equal to its weight divided by the density of that material (the latter being the product of the unit weight of water and the specific gravity of the material)

Step .-8 Adjustments for aggregate moisture -- The aggregate quantities actually to be weighed out for the concrete must allow for moisture in the aggregates. Generally, the aggregates will be moist and their dry weights should be increased by the percentage of water they contain, both absorbed and surface. The mixing water added to the batch must be reduced by an amount equal to the free moisture contributed by the aggregate -- i.e., total moisture minus absorption.

Step .-9 Trial batch adjustments -- The calculated mixture proportions should be checked by means of trial batches prepared and tested in accordance with ASTM C 192 or full-sized field batches. Only sufficient water should be used to produce the required slump regardless of the amount assumed in selecting the trial proportions.

Example 1

Find the masses of the components on a wet basis.

Table 2.7 Concrete specifications and data

Require average compressive strength	240 kg/cm^2
Slump	75 to 100 mm
Maximum size of coarse aggregate	37.5 mm
Dry-rodded mass of coarse aggregate	1600 kg/m3
Bulk specific gravity of coarse aggregate	2.68
Absorption of coarse aggregate	0.5 %
Bulk specific gravity of fine aggregate	2.64
Absorption of fine aggregate	0.7 %
Fineness modulus	2.8
Cement type I specific gravity	3.15
Total moisture of coarse aggregate	2%
Total moisture of fine aggregate	7%

Step 1. Choice of slump

Step 1. - Slump= 75 to 100 mm

Step 2. Choice of maximum size of aggregate

Step 2. - Maximum size of course aggregate= 37.5 mm

Step 3. Estimation of mixing water and air content

Step 3. – Quantity of water 181 kg/m2

Step 4. Selection of water-cement or water cementitious materials ratio

Step 4. - Water cement ratio;

$$wcr = (.69 - .61) = .08 \rightarrow . \frac{08}{.5} = .016 \rightarrow .61 + .016 = \ .626$$

Step 5. Calculation of cement content

Step 5. – Cement;

$$c = \frac{w}{w/_{l}} = \frac{181}{.626} = 289\ kg$$

Step 6. Estimation of coarse aggregate content

Step 6. – coarse aggregate;

Volume of gravel= .71 m3

Weight of gravel: $wg = (1600)(.71) = 1{,}136\ kg$

Step 7. Estimation of fine aggregate content

Step 7.- sand (fwc)= 2410 kg

Weight method: $ws = fwc - (w + c + wg)$

$$ws = 2410 - (181 + 289 + 1136)$$

$$ws = 804\ kg$$

Absolute volume method;

Water $w = \frac{181}{1000} = \ .181\ m3$

Cement = $vc = \frac{289}{(3.15)(1000)} = \ .09\ m3$

Gravel= $vg = \frac{1136}{(2.68)(1000)} = \ .42\ m3$

Air = $va = (1)(.001) = \ .001\ m3$

V without sand= $.181 + .09 + .42 + .01 = .701\ m3$

Sand= $1 - .701 = \ .299\ m3$

$$ws = (2.64)(.299)(1000) = 789.36\ kg$$

Step 8. Adjustments for aggregate moisture

Step 8. - Adjustments for moisture;

Total moisture of coarse aggregate = 2%

Total moisture of fine aggregate= 7%

Gravel=$gw = 1136(1.02) = 1{,}158.72\ kg$

Sand =$sw = 789(1.07) = 844.23\ kg$

$$wf = 181 - (1136 * 0.015) - (789 * .063)$$

$$wf = 114.25\ kg$$

Chapter II

Failure limit states

II. FAILURE LIMIT STATES

SYNOPSIS

Failure Limit States

The limit states refers the last condition that the structural can be supported by the design , so the failure being defined as any state from the structure that makes the design unsafe to build, if you are in this limit you should make the pertinences to design a safe structure.

The general from for the limit states it's consider when the capacity is bigger than the demand, so the structural limit tend to fall in two categories: strength and serviceability.

The strength based on the potential for the steel members, the failure may be have a permanent deformations or rupture, so that means that the nominal strength would be bigger or equal than required strength.

The requires strength is the internal forces that you design form the structure, are the maximum values that the beam can be resist and this one predicted the capacity from the beam.

The serviceability limit states are those conditions that need fewer requirements than the strength in the structure, the most serviceability limit states are slenderness, deflection, clearance, and vibration, and these one doesn't put people's lives in danger, neither for the structure.

Flexural Analysis and design of rectangular beams

Figure 3.1 shows the deformation and stress state on a cross section of a beam bearing bending moments alone.

All forces in the diagram (compression and tension) can be determined from three non-dimensional parameters β1, β2 y β3, where:

- β1 denotes the relationship between the mean and maximum stresses on the compression zone
- β2 places the position of the compression force

- β3 relates the maximum flexural stress with the strength of the test probes on a 28-days term

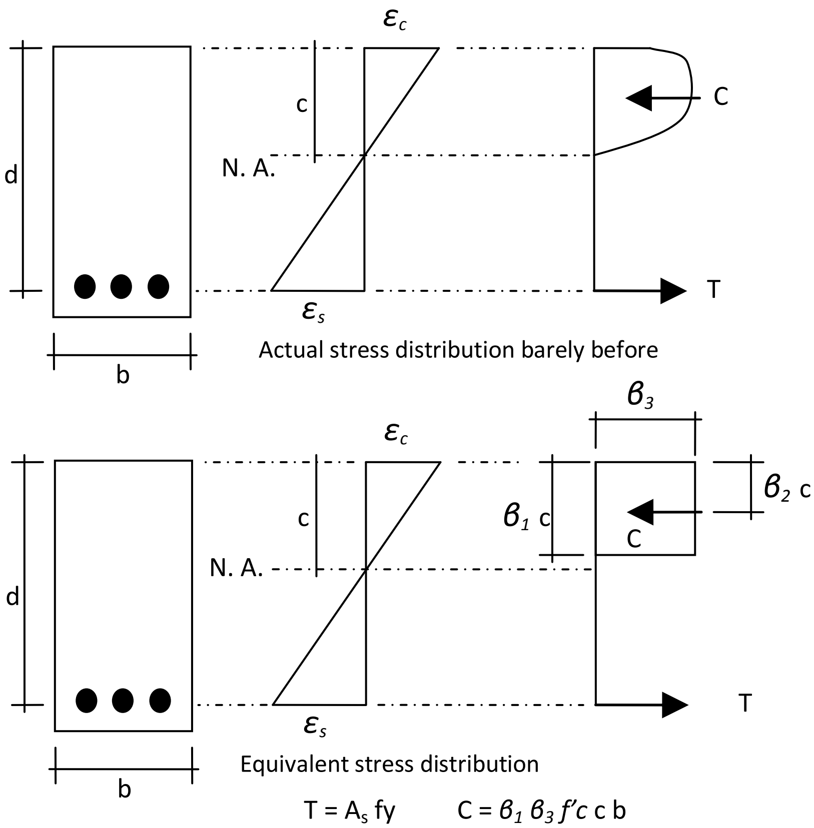


Figure 3.1 Deformation and stress distribution on a cross section of a beam subjected to flexural stresses

<u>Fundamental assumptions related to flexure axial loading and flexure plus bending NTC-2004</u>

The determination of an element strength bearing flexural forces, axial loading and a combination of both will be determined from the following assumptions:

1) A cross section that was plane before loading remains plane under load. This means that the unit strains in a beam above and below neutral axis are proportional to the distance from that axis.

2) Perfect bonding exists between concrete and steel at the interface, so that no slip can occur between the two materials.

3) Concrete is not capable to resist any tension stress whatever.

4) The unit strain of concrete in compression is 0.003; and

5) The compression stress distribution on concrete, when the maximum stress state is reached, is uniform with a value of fc'' equal to 0.85fc* for a deep equal to "a"

$\beta 1 = 0.85$; if $fc^* \leq 280$ kg/cm2

$$\beta_1 = 1.05 - \frac{fc^*}{1400} \geq 0.65; \quad \text{if } fc^* > 280 \text{ kg/cm}^2$$

"c" deep of neutral axis measured from the upper part at compression

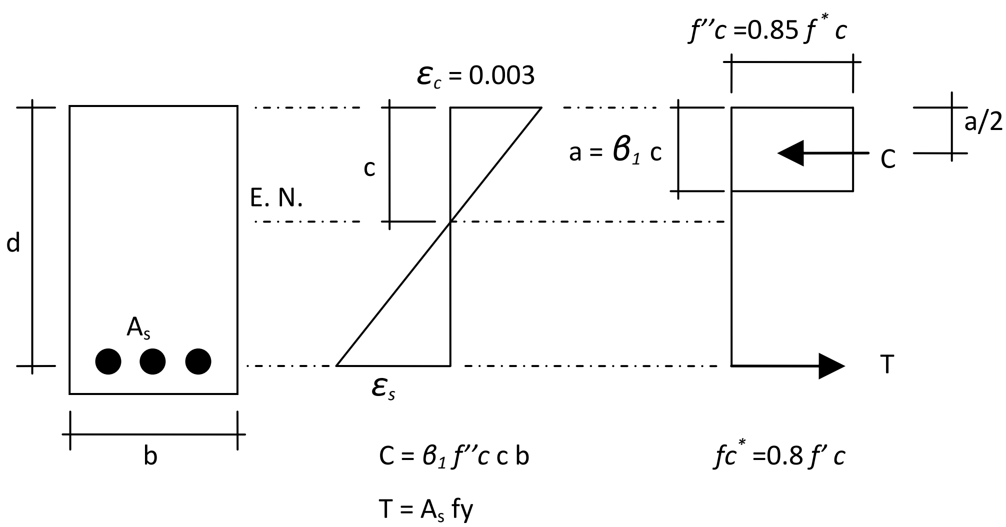


Figure 3.2 NTC-2004 Stress and strain distribution

Example 2

Find the flexural strength of the beam

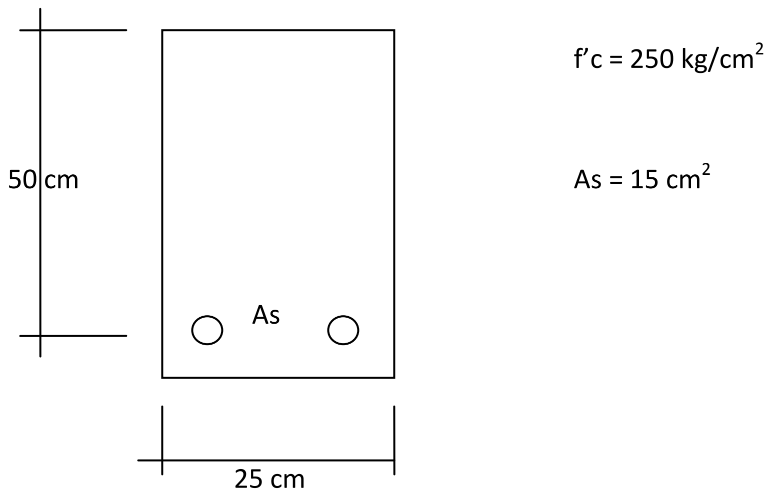


Figure 3.4

General procedure of solution:

We must consider that all the internal forces inside a structural element must be in equilibrium, so the first step is consider that the internal forces of tension and compression are equal in magnitude.

The second step consist of computing "c" and "a".

The first step determines the distance from the most stressed part in compression to the neutral axis

The second one is the height of the block of compression stressed uniformly

The third one and the last step consist of computing the internal lever arm to compute the resistant moment of the beam.

Formulae:

Tension	Compression	Nominal Moment	Resistance Moment
$T = (As)(fy)$	$C = (\beta1)(f''c)(c)(b)$	$M_n = Tz$	$M_R = F_R M_n$
Resistance Moment NTC-2004			
$M_R = F_R A_s f_y d(1 - .05q)$		$q = \frac{p\, f_y}{f''c};$	$p = \frac{As}{bd};$

Solution:

STEP 1

Tension = compression (*equilibrium)*

Tension

$$T = (As)(fy)$$

$$T = (15)(4200) = 63,000\ kg$$

Compression

$$C = (\beta1)(f''c)(c)(b)$$

$$f''c = (0.8)(0.850)(250)$$

$$f''c = 170\frac{kg}{cm2}$$

$$C = (0.85)(170)(25)c$$

$$C = 3{,}612.5c$$

Equilibrium

$$T = C \longrightarrow 63,000\ kg = 3612.5\ c$$

STEP 2

Computing "c" from the last expression

$$c = \frac{63000}{3612.5} \therefore$$

$$c = 17.43\ cm$$

The internal lever arm

$$a = (\beta 1)(c) \longrightarrow a = (0.85)(17.43)$$

$$a = 14.82\ cm$$

$$z = d - \frac{a}{2}$$

$$z = 50 - \frac{14.82}{2}$$

$$z = 42.58\ cm$$

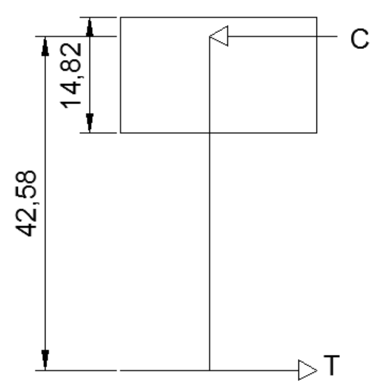


Figure 3.5

Diagram 1 for "z" and "a"

STEP 3

Let us compute the nominal moment M_n

$$M_n = Tz \longrightarrow M_n = (63{,}000)(42.58)$$

$$M_n = 26.83\ ton - m$$

Let us compute de resistances moment M_R

$$M_R = F_R M_n \longrightarrow M_R = (0.9)(26.83)$$

$$M_R = 24.14\ \ ton - m$$

$F_{R,}$ value for the NTC 2004, peg. 105, section 1.7 (Resistence factor)

Solving the problem using the espression shown in the concrete Mexican Standad in the section(NTC 2004, pag 107, section 2.2.4).

$$M_R = F_R A_s f_y d(1 - .05q)$$

$$q = \frac{p\ f_y}{f''c};$$

$$p = \frac{As}{bd};$$

$$p = \frac{15}{(25)(50)} = .012$$

$$q = \frac{(.012)(4200)}{170} = .2964$$

$$M_R = (0.9)(15)(4200)(50)\big(1 - .05(.2964)\big)$$

$$M_R = 24.14\ ton - m$$

See Problem I from the annex, page 222

NTC-2004 expressions for flexural design

$$M_R = F_R b d^2 f_c" q(1-0.5q)$$; or

$$M_R = F_R A_s fyd(1-0.5q)$$ where

$$q = \frac{pfy}{f_c"}$$

$$p = \frac{A_s}{bd}$$ (reinforcement ratio in tension)

Where *fc" are*

*fc"=0.85fc**

fc=0.85fc´*

Tension reinforced beams

According to the amount of steel to reinforce a beam, it may reach or not its yielding point before the maximum flexural loading is reached

Tension failure (Ductile failure)

Tension failure is initiated by yielding of steel, typically is gradually. Distress is obvious from observing the large deflections, widening of concrete cracks associated with yielding of steel, and measures can be taken to avoid total collapse. For this case the element is called underreinforced.

Compression failure (Brittle concrete compression failure)

On the other hand, a compression failure in flexure is a sudden failure and cannot permit to take any action to avoid total collapse as no deflection or cracks can be seen clearly before failure occur. For this case the element is called overreinforced.

Balance failure

At balanced failure, the steel strain is exactly equal to εy when the strain in the concrete simultaneously reaches its crushing strain εs.

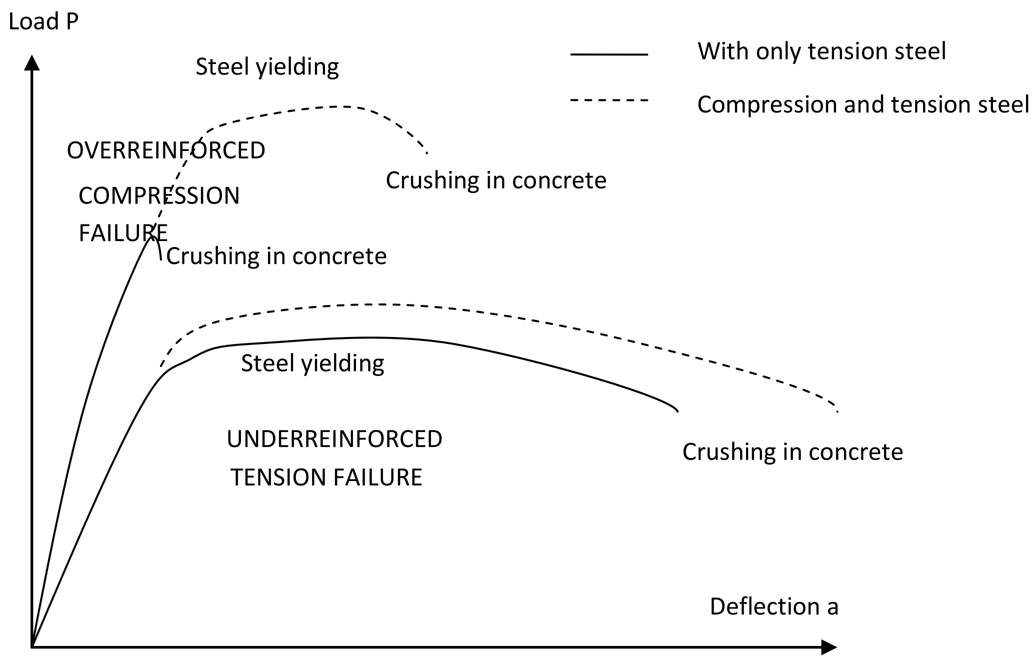


Figure 3.3 Load-deflection graph for overreinforced and underreinforced beams

Additional comments

If compression failure should occur it does not gives any warning of distress and could be catastrophic. While a tension failure is always gradual so measures can be taken to avoid failure.

Because of this a concrete reinforced beam is to be design to have a failure in tension. In practice it could be done placing an amount of steel ρ less than the one ρb for the balanced failure. This is regulated by the construction codes for the following reasons:

1. For a beam with ρ exactly equal to ρb the compressive strain limit of the concrete would be reached, theoretically, at precisely the same moment

that the steel reaches its yield stress, without significant yielding before failure.

2. Material properties are never known precisely.

3. The strain-hardening of the steel, not accounted for in the design, may lead to a brittle compression failure.

4. The actual steel area provided, considering standard reinforcing bars sizes, will be always equal or larger than required, tending toward overreinforcement.

5. The extra ductility provided by beams with lower values of ρb increases the deflection capability and thus, provides warning prior to failure.

Maximum reinforcement NTC-2004

The maximum steel area for elements that will not resist earthquake the amount of steel will be 90 percent of that of the balanced failure

Concrete sections with only tension steel have a balanced failure when its steel is equal to:

$$A_{s,bal} = \frac{fc"}{fy}\frac{6000\beta_1}{fy+6000}bd$$ Steel for balanced failure

Where *fc" are*

$$fc"=0.85fc^*$$

$$fc^*=0.85fc'$$

So the maximum steel area will be:

$$A_{s,\max} = 0.9A_{s,bal} = 0.9\frac{fc"}{fy}\frac{6000\beta_1}{fy+6000}bd$$

Where b and d are width and effective deep of the section reduced in 20 mm. Such reduction is not necessary for dimensions larger than 200 mm, nor in elements where precaution are taken to guarantee that the dimension to resist the forces will be not less than that being projected.

Minimum reinforcement

The minimum reinforcement in tension will be at least the one that provides a resistant moment equal to 1.5 the cracking moment. To evaluate the area required the following formula can be used:

$$A_{s,\min} = \frac{0.7\sqrt{f'c}}{fy} bd$$

Where “b” and “d” are the width and the effective deep, respective

Rectangular Beams with Tension and Compression Reinforcement

The essential difference between tension and tension plus compression reinforced beams is that the latter has compression steel (Figure 4.1)

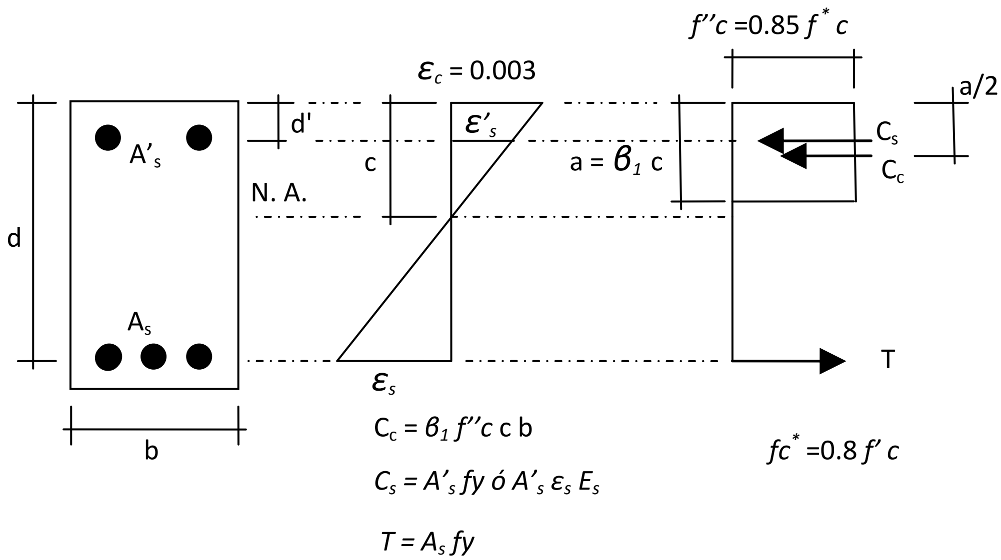


Figure 4.1 NTC-2004 Stress and strain distribution for a beam with tension plus compression steel

In order to determine the strength of a beam with tension and compression steel the easiest procedure is trial and error.

Generally, the moment capacity of a member with both steels is determined in a similar fashion than for a tension reinforced member, but considering now that the compression force is now the sum of concrete (Cc) plus steel (Cs).

To determine Cs it is necessary to compute the strain (εs) at compression steel level and from that strain the steel stress (f's) can be computed. This value could be less o larger than the yielding strain.

So it is necessary to distinguish two cases when compression steel is yielding and when it is not yielding. It is useful to initiate the calculations thinking that the compression steel in yielding.

$\varepsilon'_s > \varepsilon_y$; steel compression is yielding

$\varepsilon'_s < \varepsilon_y$; steel compression is not yielding

$$\varepsilon_y = \frac{fy}{E_s} = \frac{4200}{2x10^6} = 0.0021$$

, then fs = εs Es

Where:

εs unit strain for compression steel

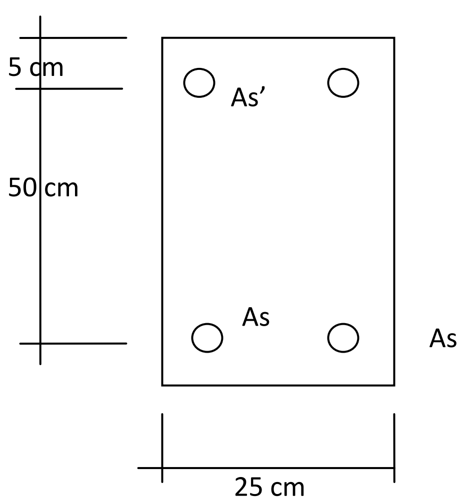
εy steel unit strain for which yield starts

fy steel yielding stress (4200 kg/cm2)

Es modulus of elasticity of steel (2 x 106 kg/cm2)

Example 3

Find MR of the reinforced concrete section



f’c = 250 kg/cm^2

As = 10 cm^2

As’ = 2.5 cm^2

Figure 4.3

General procedure solution:

Like the last example; we must consider that all the internal forces inside a structural element must be in equilibrium, so the first step is consider that the internal forces of tension and compression are equal in magnitude.

The second step is decrease “c” and depends of the own criteria, one of these could be reducing in 10% of the first “c”.

The Third step then we needs to compute the unit strain for compression steel ($\in' s$) in order to compute the compression force for the steel.

The fourth step computes the compression force for the concrete.

The fifth step accumulates the two compression forces in order to compare with the tension force.

The Sixth step calculates the tension force, and then we compare the compression and tension force. If the tension is bigger than the compression, we need to propose and other “c”, and so again the last steps, almost to find that compression is similar to tension o bigger, when this happened calculate M_R.

Formulae:

Tension = compression $A_S f_y = \beta c b f''_c$	$\in' s = \frac{(c-5)(\in_C)}{C}$	$f's = Es \in 's$	$C's = f's\, As$
$a = \beta c$	$cc = abf''c$	$T = fy\, As$	$M_R = (F_R)(M_N)$
Moment of concrete	Moment for steel	Moment for tension	Nominal Moment $M_N = M_{cc} + M_{c's} + M_t$
$M_{cc} = C_c\left(c - \frac{a}{2}\right)$	$M_{c's} = C'_s(c - 0.05)$	$M_t = T(d-c)$	

Solution:

STEP 1

Tension = compression

$$A_S\, f_y = \beta c b f''_c$$

$$c = \frac{A_S\, f_y}{\beta b f''_c}$$

$$c = \frac{(10)(42{,}000)}{(.85)(25)(170)} = 11.6\ cm$$

STEP 2

The first guess is reducing “c”, so it´s optional and depends of own criteria.

First guess

C=9

STEP 3

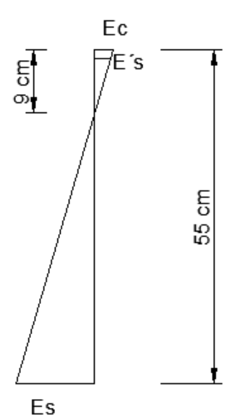


Figure 4.4

$$\in' s = \frac{(c-5)(\in_C)}{C}$$

$$\in' s = \frac{(9-5)(0.003)}{9} = 0.0013$$

Steel is not yielding

$$f's = Es \ \in 's$$

$$f's = (2x10^6)(.0013) = 2600 \ kg/cm2$$

$$C's = f's \ As$$

$$C's = (2600)(2.5) = 6.5 \ ton.$$

STEP 4

$$fs = fy = 4{,}200 \frac{kg}{cm2}$$

$$a = \beta c = (.85)(9) = 7.65$$

$$cc = abf''c$$

$$cc = (7.65)(25)(170) = 32.5 \ ton$$

STEP 5

$$C = 32.5 + 6.5 = 39 \ ton$$

STEP 6

$$T = fy \ As$$

$$T = (4{,}200)(10) = 42 \ ton$$

Comparing Tension and compression we can see that tension y bigger than compression so we need to propose another “c”.

C= 9.7 cm

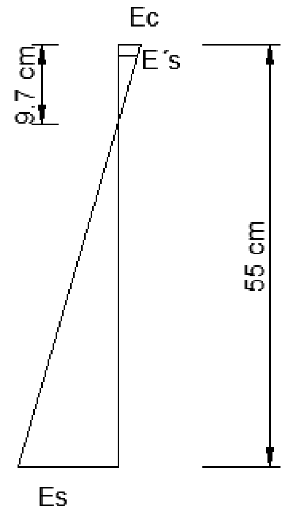


Figure 4.5

$$\in' s = \frac{(9.7-5)(0.003)}{9.7} = 0.00145$$

$$f's = (2x10^6)(.00145) = 2900\ kg/cm2$$

$$C's = (2900)(2.5) = 7.2\ ton.$$

$$fs = fy = 4{,}200\frac{kg}{cm2}$$

$$a = \beta c = (.85)(9.7) = 8.24$$

$$cc = (8.24)(25)(170) = 35.041\ ton$$

$$C = 35.041 + 7.2 = 42.2\ ton$$

$$T = (4{,}200)(10) = 42\ ton$$

$$T \approx C$$

So let us compute M_N

Moment of concrete compression force

$$M_{cc} = C_c\left(c - \frac{a}{2}\right)$$

$$M_{cc} = 35.04\left(.097 - \frac{.084}{2}\right) = 1.96\ ton - m$$

Moment for steel compression force

$$M_{c's} = C'_s(c - 0.05)$$

$$M_{c's} = 7.268(.097 - 0.05) = .3415\ ton - m$$

Moment for tension force

$$M_t = T(d - c)$$

$$M_t = 42(.55 - .097) = 19.02\ ton - m$$

Nominal moment

$$M_N = M_{cc} + M_{c's} + M_t$$

$$M_N = 1.96 + .3415 + 19.02 = 21.32\ ton - m$$

Resistance moment

$$M_R = (F_R)(M_N)$$

$$M_R = (.9)(21.32) = 19.18\ ton - m$$

See Problem II from the annex, page 223

Formula from the NTC-2004

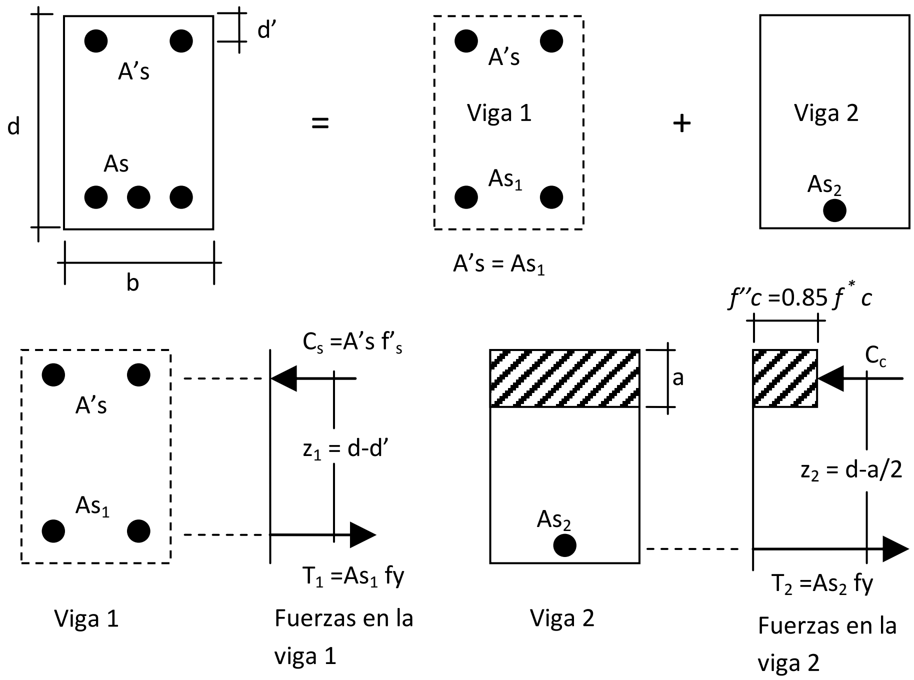


Figure 4.2 Nominal resistant moment for beams with compression and tension steel. NTC 2004.

$$M_R = F_R\left[(A_s - A'_s)\,fy\left(d - \frac{a}{2}\right) + A'_s\ fy(d - d')\right]$$

Where:

$$a = \frac{(A_s - A'_s)\,fy}{f_c"b}$$

a = deep of equivalent block of stresses

As = steel area in tension

A's = steel area in compression

d' = distance between compression steel center and the farthest part in compression

This equation is only valid when the compression steel is yielding. This is true when:

$$p - p' \geq \frac{6000\beta_1}{6000 - fy}\frac{d'}{d}\frac{f_c"}{fy}$$

Where:

$p' = \dfrac{A'_s}{bd}$ reinforcement ratio in compression

When this condition is false the resistant moment of a section shall be calculated from the equilibrium analysis like in the example.

Flexural analysis and design of "T" beams

With exception of precast systems, reinforced concrete floors, roofs, slabs, etc., are almost always monolithic. Almost always beams are casts at once with slabs and the beams stirrups extent into the slab. It is evident, therefore, that a part of the slab will act with the upper part of the beam to resist longitudinal compression. The resulting beam section is a T-shaped rather than rectangular. The slab forms the beam flange, while the part of the beam projecting below the slab forms what is called the web or stem.

Effective flange width

In Figure 5.1a, it is evident that if the flange is but little wider than the web width, the entire flange can be considered effective in resisting compression. For the floor system shown in Figure 5.1b, it may be equally obvious that elements of the flange midway between the beam webs are less highly stressed in longitudinal compression than those elements directly over the web. So it is convenient in design to make use of an effective flange width, which may be smaller, than the actual flange width but is considered to be uniformly stressed at maximum value. This effective width has been found to depend primarily on the beam span and on the relative thickness of the slab.

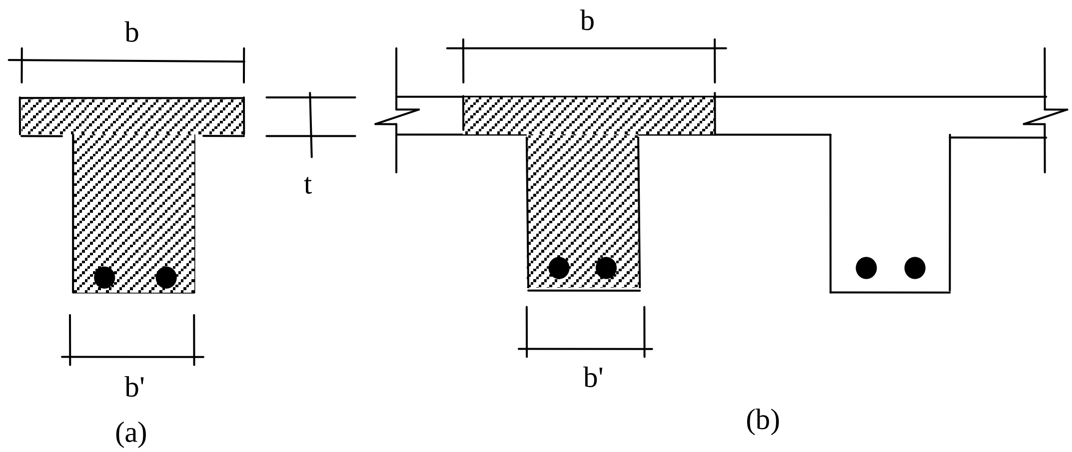


Figure 5.1 Effective flange width of T beams

The criteria for the effective flange given in NTC-2004 are as follow:

For L and T sections the overhanging slab width on either side of the web will be the less of the three values:

- One-eighth of the span length of the beam minus half width of the web.
- Half of the distance between edges of webs.
- Eight times the flange depth.

Strength analysis

The neutral axis of a T beam may be either in the flange or in the web, depending upon the proportions of the cross section, the amount of tensile steel, and the strengths of the materials.

If the calculated depth to the neutral axis is less than or equal to the slab thickness t, the beam can be analyzed as if it were a rectangular beam of width equal to b, the effective flange width Figure 5.2a.

When the neutral axis is in the web, as in Figure 5.2b, in this case an analysis of stress-strains must be carried out to know the actual moment capacity of the beam.

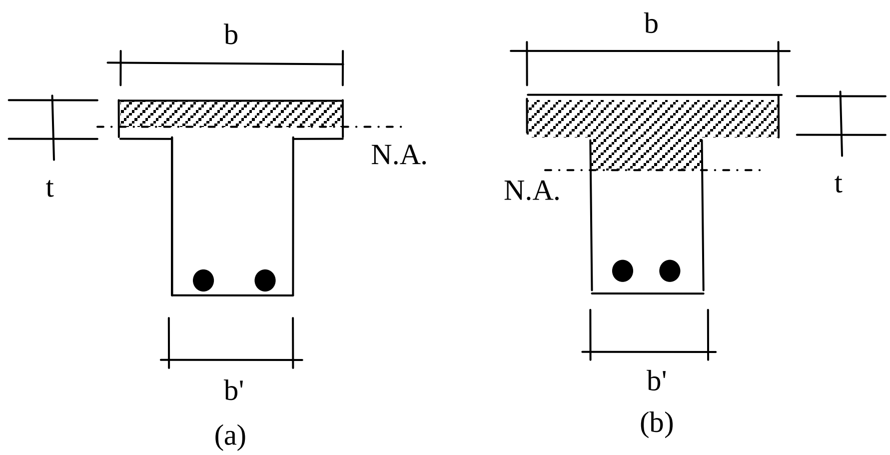


Figure 5.2 Effective cross section of T beams

In treating T beams, it is convenient to adopt the same equivalent stress distribution that is used for beams of rectangular cross section. The rectangular stress block, having a uniform compressive-stress intensity, f’’c (Figure 5.3).

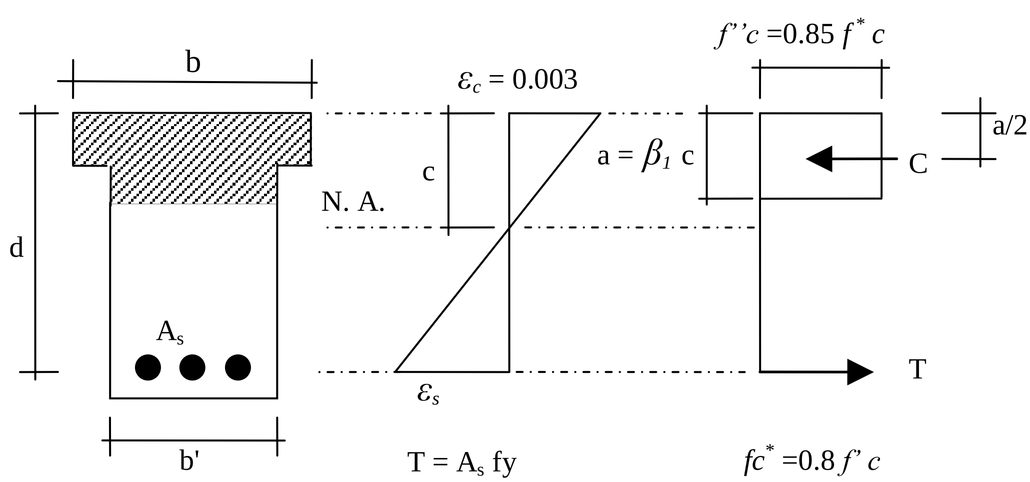


Figure 5.3 Strain and equivalent stresses for T beams

The depth of the compressive block a can be calculated with the following expression.

$$a = \frac{A_s fy}{f_c'' b}$$

If a is less than or equal to the flange thickness t, the member may be treated as a rectangular beam of width b and depth d.

If a is greater than t, the required analysis is as follows.

It will be assumed than the strength of the T beam is controlled by yielding of the tensile steel. This will always be the case because of the large compressive concrete area provided by the flange.

In addition an upper limit can be established for the reinforcement ratio to ensure that it is so, as will be shown.

As a computational device, it is convenient to divide the total tensile steel into two parts.

The first part, Asp, represents the steel which, when stress to fy, is required to balance the longitudinal compressive force in the overhanging portions of the flange that are stressed uniformly. Thus,

$$A_{sp} = \frac{f_c''(b-b')t}{fy}$$

The force Asp fy an the equal and opposite force fc” (b-b’)t act with the lever arm d-(t/2) to provide the nominal resisting moment

$$M_{n1} = A_{sp} fy\left(d - \frac{t}{2}\right)$$

The remaining steel area, A_s-A_{sp}, at the stress *fy*, is balanced by the compression in the rectangular portion of the beam. The depth of the equivalent rectangular stress block in this zone is found from the horizontal equilibrium:

$$a = \frac{(A_s - A_{sp})\,fy}{f_c''b'}$$

An additional moment M_{n2} is thus provided by the forces $(A_s$-$A_{sp})fy$ and $af_c''b'$ acting at the lever arm *d-(a/2)*:

$$M_{n2} = (A_s - A_{sp})\,fy\left(d - \frac{a}{2}\right)$$

And the total nominal resisting moment is the sum of the two parts:

$$M_n = \left[A_{sp}\,fy\left(d - \frac{t}{2}\right) + (A_s - A_{sp})\,fy\left(d - \frac{a}{2}\right)\right]$$

To finally find the design resistant moment a reduction factor needs to be applied to the presiding expression:

$$M_R = F_R\left[A_{sp}\,fy\left(d - \frac{t}{2}\right) + (A_s - A_{sp})\,fy\left(d - \frac{a}{2}\right)\right]$$

The applied of the presiding equation needs the following expression to be true. (Tensile steel is at *fy* stress)

$$A_s \leq \frac{fc''}{fy}\frac{6000\beta_1}{fy + 6000}b'd + A_{sp}$$

Example 4

Find the flexural strength of the T-beam for the two cases:

a) As = 11.8 cm^2 and

b) As = 25 cm^2

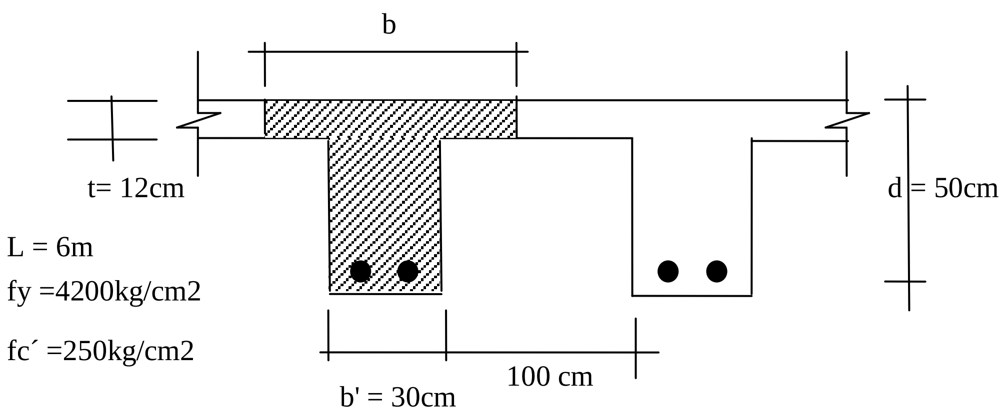


Figure 5.4

General procedure for the solution:

The first step, we need to compute the overhanging slab width "b" in order to choose the smallest of the three formulas that we have.

One-eighth of the span length of the beam minus half width of the web.

$$b_1 = \frac{L}{8} - \frac{b'}{2}$$

Half of the distance between edges of webs.

$$b_2 = \frac{Distance}{2}$$

Eight times the flange depth.

$$b_3 = (8)(t_w)$$

This is from NTCDF code, page 106-107, section 2.2.3.

The second step, consist of calculated, the depth of the compressive block "a". And make the correct analysis.

The third step consist of dividing the total steel A_s into two parts A_{SP} which balance the compressive force in the overhanging part of the T beam and ($A_{SP} - A_s$) which balance the compressive force in the central part of the T beam, and at the same time the moment is computed.

The Fourth step consist of determining the moments due to the two parts of steel and then to sum both moments to find the total moment.

*note

The applied of the presiding equation needs the following expression to be true. (Tensile steel is at *fy* stress)

$$A_s \leq \frac{fc"}{fy}\frac{6000\beta_1}{fy+6000}b'd + A_{sp}$$

Formulae:

Depth $a = \frac{As\,fy}{f''cb}$;	Tensile steel $A_{sp} = \frac{f''c(b-b')t}{fy}$	Nominal resisting moment $M_{n1} = A_{sp}\,fy\,\left(d - \frac{t}{2}\right)$;	Additional moment $M_{n2} = \left(A_s - A_{sp}\right)fy\,\left(d - \frac{a}{2}\right)$
Nominal resisting moment $M_n = M_{n1} + M_{n2}$	Resistant moment $M_R = F_R M_n$		

STEP 1

Overhanging

One-eighth of the span length of the beam minus half width of the web

$$b_1 = \frac{6}{8} - \frac{0.30}{2} = 0.6\ m = 60\ cm$$

Half of the distance between edges of webs

$$b_2 = \frac{100}{2} = 50\ cm$$

Eight times the flange depth.

$$b_3 = (8)(12) = 96\ cm$$

STEP 2

Depth of the compressive block

$$a = \frac{As\ fy}{f''cb};$$

With:

$$As_1 = 11.8$$

$$a = \frac{(11.8)(4{,}200)}{(170)(50)} = 5.83\ cm$$

For this cases note that the compressive block of concrete is within the slab, so the resisting moment is determined as we do for rectangular beams.

And:

$$As_2 = 25$$

$$a = \frac{(25)(4{,}200)}{(170)(50)} = 12.35\ cm$$

In this case the neutral axis is out of the slab so the resisting moment is determined as follows

STEP 3

Divide the steel into two parts and computing the corresponding moments.

The tensile steel, A_{sp},

$$A_{sp} = \frac{f''c(b - b')t}{fy};$$

$$A_{sp} = \frac{(170)(50 - 30)(12)}{4{,}200} = 9.71\ cm2$$

Now we compute the nominal resisting, moment, M_{n1}

$$M_{n1} = A_{sp}\,fy\,\left(d - \frac{t}{2}\right);$$

$$M_{n1} = (9.71)(4200)\left(50 - \frac{12}{2}\right);$$

$$M_{n1} = 17.94\ ton - m$$

The depth of the equivalent rectangular stress block

$$a = \frac{(As - Asp)\,fy}{(f''c\,)(b')};$$

$$a = \frac{(25 - 9.71)4{,}200}{(170)(30)} = 12.59\ cm$$

Now we compute the additional moment

$$M_{n2} = \left(A_s - A_{sp}\right)fy\,\left(d - \frac{a}{2}\right);$$

$$M_{n2} = (25 - 9.71)(4{,}200)\left(50 - \frac{12.59}{2}\right);$$

$$M_{n2} = 28.06\ ton - m$$

STEP 4

The nominal resisting moment

$$M_n = M_{n1} + M_{n2}$$

$$M_n = 17.94 + 28.06 = 46\ ton - m$$

The resistant moment

$$M_R = F_R M_n$$

$$M_R = (0.9)(46) = 41.4\ ton - m$$

See Problem III from annex, page 224

Shear and Diagonal Tension in Beams

Beams must also have an adequate safety margin against other type of failure, some of which is more dangerous than flexural failure. This may be so because of the uncertainty in predicting certain other modes of collapse, or because of the catastrophic nature of some other types of failure.

Shear failure is one example, it is difficult to predict accurately, in spite of many decades of experimental research and the use of highly sophisticated analytical tools.

As tension strength of concrete is low compared to compressive resistant, a concrete beam will tend to fail perpendicularly to main stresses surfaces in tension. So, in beams it is necessary to place steel to increase its low tension strength.

The shear stress in most beams is far below the direct shear strength of the concrete. The real concern is with diagonal tension stress, resulting from the combination of shear stress and longitudinal flexural stress (Figure 6.1).

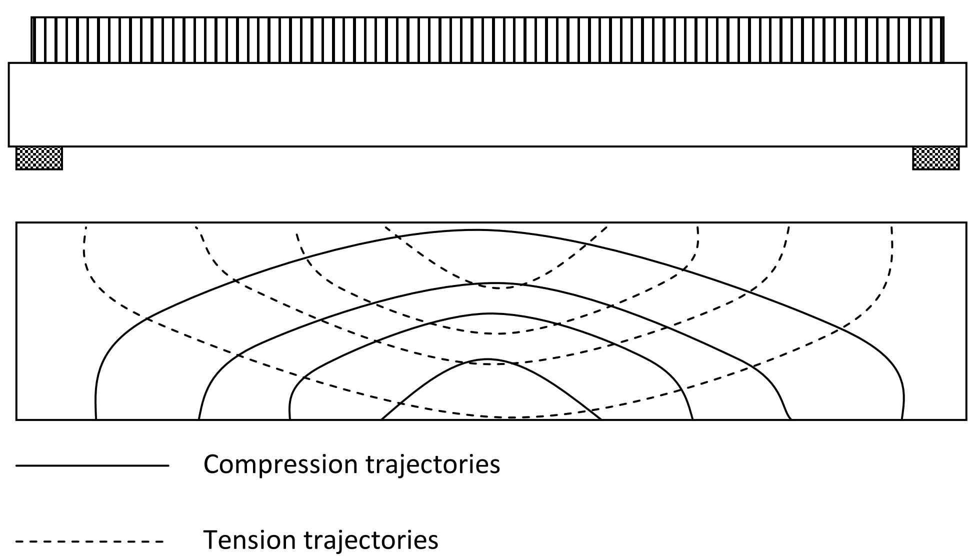


Figure 6.1 Main stresses trajectories

Shear failure of reinforced concrete, is more properly called diagonal tension failure.

From this reason it could be concluded that a reasonable way to reinforce beams for diagonal tension is following the trajectories of tension stresses. This is not a practical way, though.

Reinforced concrete beams without shear reinforcement

Diagonal tension failure

When a diagonal crack (tension crack) appears and goes along all the beam depth and a sudden failure occurs it is said that the element fails in diagonal tension. This would result is a catastrophic failure without warning. So it is needed to provide web reinforcement to avoid this type of failure.

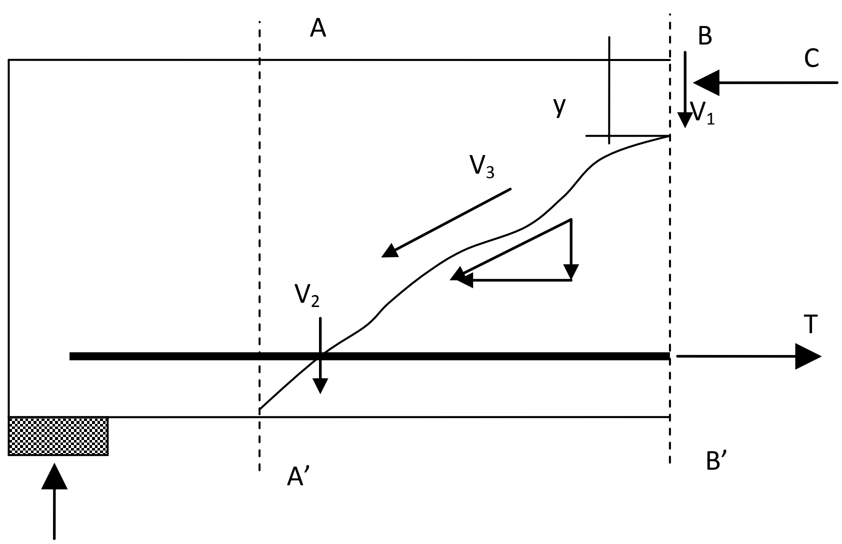


Figure 6.2 Forces at a diagonal crack in a beam without web reinforcement

Reinforced concrete beams with web reinforcement

The behavior of beams with web reinforcement in diagonal tension is similar to the behavior of beams without web reinforcement until the first diagonal crack appears;

from this moment on the presence of web reinforcement limits the growth of diagonal cracks. Whatever the web reinforcement it never adds any resistance to the element until the very moment when the cracks form.

If the element has enough web reinforcement the cracks will be very small, and the beam failure will be by flexural stresses.

So, the shear resistance of a beam is carried out by two components: concrete and web reinforcement

$$V_n = V_c + V_s$$

Where:

Vn = Nominal strength of a element with web reinforcement

Vc = Concrete resistance to shear

Vs = Web reinforcement resistance to shear

In design it is always wanted enough web reinforcement so the failure will occurs by flexural effects and not by shear.

Important notes

In order to have effective web reinforcement, the stirrups should be placed at a space that any diagonal tension could be intercepted by at least one stirrup.

The web reinforcement increases the element ductility as long as it provides lateral confinement to the concrete.

Evaluation of the strength participation of web reinforcement

Truss analogy

This analogy gives a quantitative explanation for a beam with web reinforcement, though it does not mach actual values.

Consider an element that is already cracked, due to diagonal tension, the web reinforcement could be taken as a tension elements, the concrete between cracks as a compression struts and the compression concrete zone as the elements in compression (Figure 6.3).

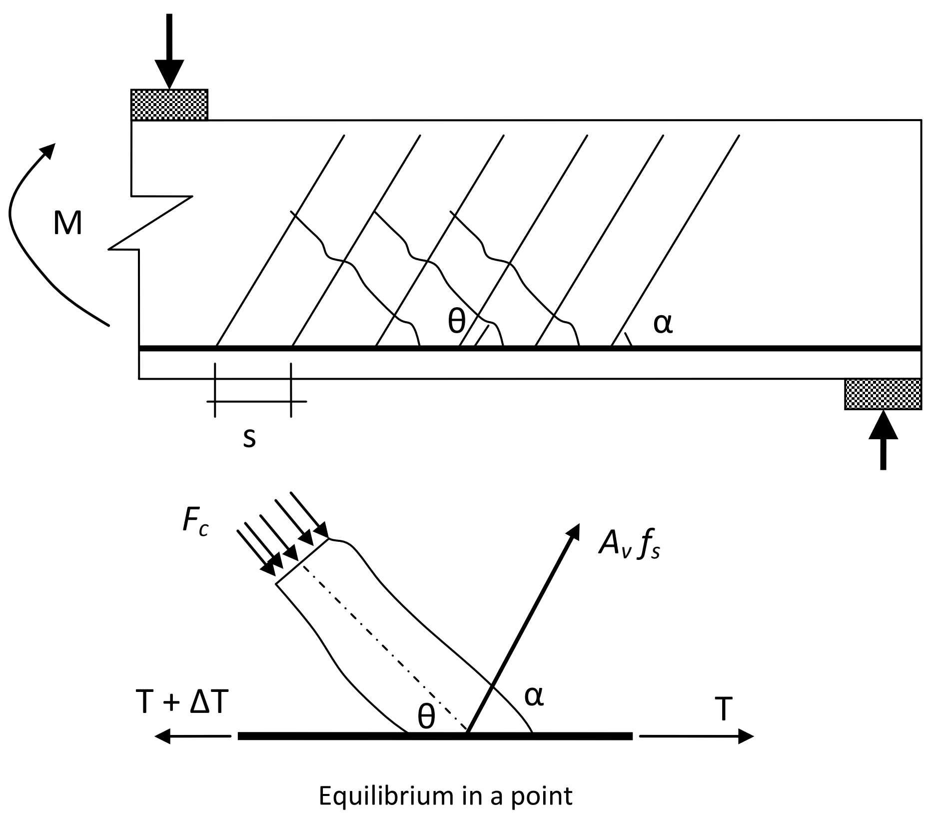


Figure 6.3 Truss analogy

Within the analysis it is supposed that the cracks and the beam axis are at an angle θ, and the web reinforcement with the beam axis are at an angle α. The assumptions are as follows:

- The compression zone only takes normal compressive stresses.
- The longitudinal reinforcement only takes normal tension stresses.
- All diagonal tension is resisted by the web reinforcement.
- Cracks grow from the longitudinal reinforcement to the center of the compression zone.
- The weight of the beam is not taken into account. The moment gradient between two sections separated s is equal to Vs, where V is the shear force between the two sections.

From the Y equilibrium

$$A_v f_s sen\alpha = F_c sen\theta \quad (1)$$

From the X equilibrium

$$\Delta T = A_v f_s \cos\alpha + F_c \cos\theta \quad (2)$$

From assumption five

$$\Delta T = \frac{\Delta M}{z} = \frac{Vs}{z} \quad (3)$$

Substituting Fc from Eq. 1 and ΔT from Eq. 3 in Eq. 2:

$$\frac{Vs}{z} = A_v f_s \left[\cos\alpha + \frac{sen\alpha}{\tan\theta} \right]$$

So the shear strength resisted in one section of web reinforcement Av is:

$$V = \frac{A_v f_s z}{s} \left[\cos\alpha + \frac{sen\alpha}{\tan\theta} \right]$$

First, taking into account that the cracks generally form a θ of 45º, it yields:

$$V = \frac{A_v f_s z}{s} \left[sen\alpha + \cos\alpha \right]$$

Now, taking into account that the web reinforcement is placed generally at a α of 90º, it yields:

$$V = \frac{A_v f_s z}{s}$$

Finally, in beams the internal lever arm z can be replaced by the beam effective depth d, it yields:

$$V = \frac{A_v f_s d}{s}$$

Shear formulae in NTC 2004

1) To evaluate the shear strength in rectangular sections

2) Non prestressed beams.

3) For beams with ratio span-deph (L/h) not less than 5.

4) If p < 0.015

$$V_{cR} = F_R bd(0.2 + 20p)\sqrt{f^*_c} \quad (1)$$

If p >= 0.015

$$V_{cR} = 0.5F_R bd\sqrt{f^*_c} \quad (2)$$

If the ratio L/h is less than 4 and the loads and reactions compress the top and bottom faces of beam, VcR will be taken form Eq. (2) multiplied by:

$$3.5 - 2.5\frac{M}{Vd} > 1.0$$

But VcR never will be more than:

$$1.5\left(F_R bd\right)\sqrt{f^*_c}$$

M and V are the moment and shear force in the section being analyzed

If the loads and reactions do not compress the top and bottom faces of beam VcR will be calculated with Eq. (2)

Minimum web reinforcement

For beams minimum web reinforcement should be provided when the design shear force Vu, is less than VcR. It will be calculated with the following expression:

$$A_{v,\min} = 0.30\sqrt{f^*_c}\,\frac{bs}{fy}$$

This reinforcement will be from bars of diameter no les s than 7.9 mm (No. 2.5) and should not be placed at a distance greater than d/2.

Placement distance of web reinforcement

When Vu is greater than VcR, the distance “s “ of stirrups will be calculated using the following expression:

$$s = \frac{F_R A_V\, fyd\,(sen\theta + \cos\theta)}{V_{sR}}$$

Where:

Av = Cross section of steel within a s distance
θ = Angle that forms the web reinforcement with the beam axis, and
VsR = Shear force that resists the web reinforcement (VsR = Vu – VcR)

The web reinforcement will never be less than the minimum specified

The distance between stirrups will never be less than 60 mm

If Vu is greater than VcR but less or equal to

$$1.5\left(F_R bd\right)\sqrt{f *_c}$$

The distance between stirrups will never be greater than

0.5d

If Vu is greater than

$$1.5\left(F_R bd\right)\sqrt{f *_c}$$

The distance between stirrups will never be greater than

0.25d

Limits for Vu in beams

Vu will never be greater than

$$2.5(F_R bd)\sqrt{f^*_c}$$

Example 5

Find the shear strength for the beam in the figure

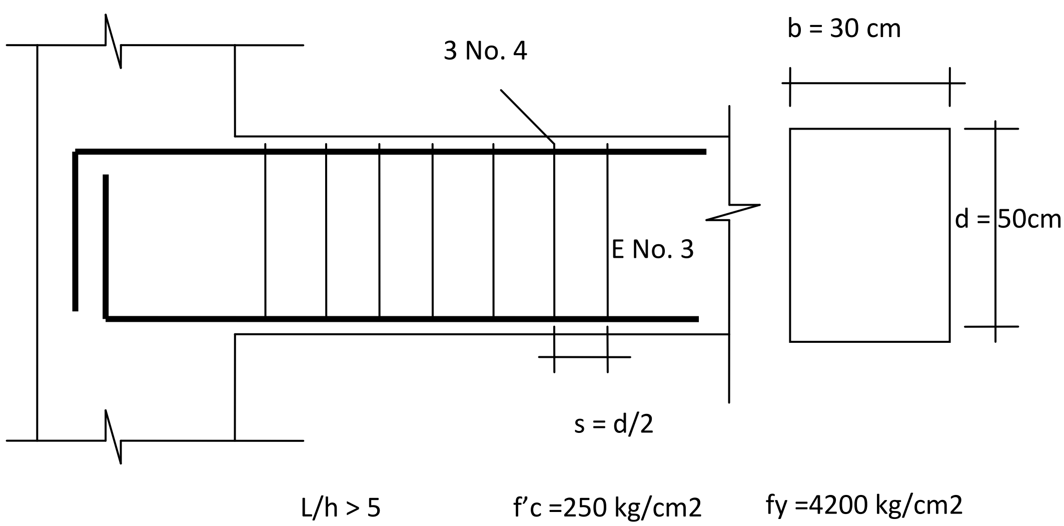


Figure 6.4

General procedure for the solution

The First step is to choose the expression to compute V_{CR}

The Second Step is to compute the web reinforcement resistance to shear

The Third Step is to calculate the shear resistance of a beam, resisted by the concrete and the web reinforcement.

Formulae

$$V_{CR} = F_R bd(0.2 + 20p)\sqrt{f *_c} \qquad V_{CR} = 0.5F_R bd\sqrt{f *_c} \qquad f *_c = 0.8(f'_c) \qquad V_s = \frac{A_v f_s d}{s}$$

$$V_n = V_{CR} + V_s \qquad p = \frac{A_s}{bh}$$

STEP 1

$$\frac{l}{h} > 5$$

Now we know that either of the two expressions are applicable depending of "p"

If p<0.015

$$V_{CR} = F_R bd(0.2 + 20p)\sqrt{f *_c} \quad (1)$$

If p>= 0.015

$$V_{CR} = 0.5F_R bd\sqrt{f *_c} \quad (2)$$

We need to compute "p" for the restrictions about the span-depth not less than 5

$$p = \frac{A_s}{bh}$$

$$A_s = (\#\ of\ bars)(Cross\ section\ of\ a\ bar)$$

$$p = \frac{(3)(1.267)}{(30)(55)} = .0023$$

A_S is p<0.015 then we use the following expression to compute V_{CR}

$$V_{CR} = F_R bd(0.2 + 20p)\sqrt{f *_c}$$

$$f *_c = 0.8(f'_c);$$

$$f *_c = 0.8(250) = 200$$

$$V_{CR} = (.7)(30)(50)(0.2 + 20(.0023)\sqrt{200}$$

$$V_{CR} = 3.65\ ton$$

STEP 2

The web reinforcement resistance to shear:

Now

$$V_s = \frac{A_v f_s d}{s}$$

$$s = \frac{d}{2} = \frac{50}{2} = 25\ cm$$

$$f_s = f_y$$

A_V= cross section of the two #3 bars, who is 1.42

$$V_s = \frac{(1.42)(4200)(50)}{25} = 11.928\ ton$$

STEP 3

So, the shear resistance is:

$$V_n = V_{CR} + V_s$$

$$V_n = 3.65 + 11.928 = 15.57\ ton$$

See Problem IV from the annex, page 225

SHORT COLUMNS

Axial Compression on Columns

Consider a concrete prism; the prism strength will tend to reduction on growing its slenderness ratio until reaching a minimum value of 85 percent of a prism of slenderness ratio equal to two. So the compression strength of a concrete element can be determined with the following expression:

$$P = 0.85 f'cA_g$$

P = Compression strength

Ag = Cross section of concrete

If longitudinal steel is added and stirrups used to maintain the first in its place the axial load strength of the element can be computed from:

$$P_n = 0.85 f'cA_g + A_s fy$$

To find the design load the introduction of certain strength reduction factor is needed.

The latter expression shows that the axial strength of a column is due to the concrete and to the steel.

Behavior of spirally and stirrup reinforced columns

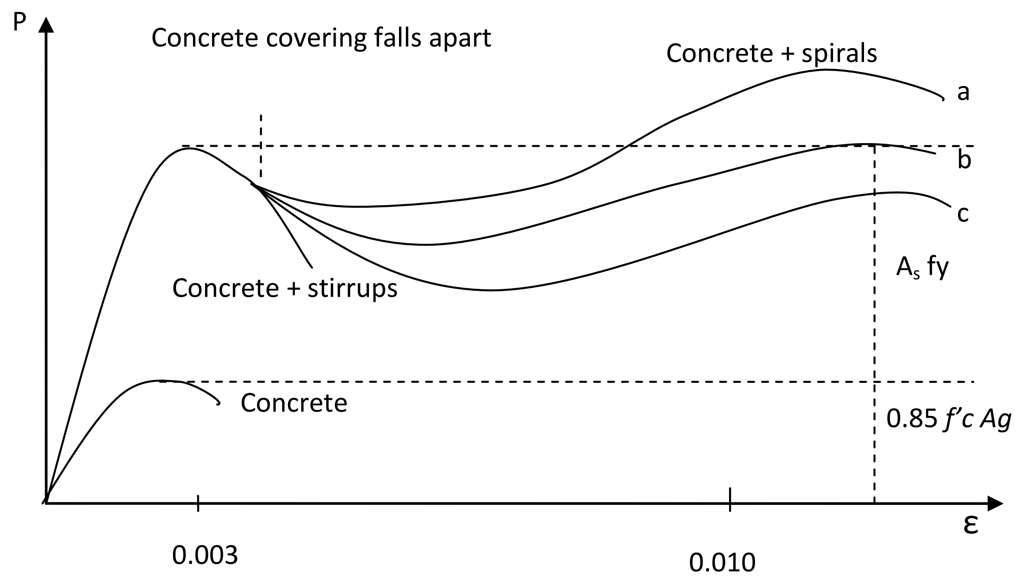


Figure 7.1 Behavior of spirally and tied columns

Watching Figure 7.1 it can be seen that spirally reinforced columns could have a second maximum (curve a) depending on the volumetric reinforcement ratio (ρs).

Four factors can be taken into account in determining axial load strength of column:

1) Core concrete area
2) Longitudinal steel
3) Concrete outside stirrups or spirals
4) Transverse reinforcement

Note: 3 and 4 factors are never considered acting simultaneously.

Evaluating the contribution to resist axial loads of transverse reinforcement

It is possible to evaluate the contribution of transverse reinforcement as a function of volumetric reinforcement ratio and the mechanical properties of steel.

$$\rho_s = \frac{\text{Steel volume of one spiral}}{\text{Core concrete volume within one spiral}}$$

$$\rho_s = \frac{\pi d A_e}{\frac{\pi d^2}{4} s} = \frac{4A_e}{sd}$$

Where :

d = diameter on core concrete measure from outside faces of steel

Ae = cross area of one spiral

s = distance between spirals

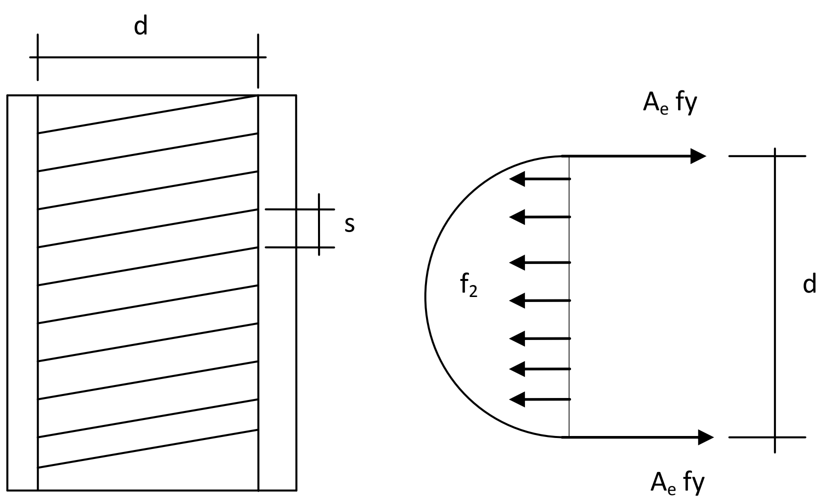


Figure 7.2 Strength contribution of the shell

From the equilibrium of Figure 2

$$2A_e fy = f_2 sd$$

$$f_2 = \frac{2A_e fy}{sd}$$

Taking into account the volumetric reinforcement ratio

$$f_2 = \frac{\rho_s fy}{2}$$

It is to note that f2 is acting in a perpendicular plane to that in which the axial load is acting. Experimentally it has been determined that the strength contribution of the spirals in a parallel direction of the axial load is:

4.1f2Ac.

Where Ac is the core concrete area

So the strength contribution of the shell (transverse reinforcement) will be, approximately:

$$P_{\rho s} = 2\rho_s fyA_c$$

Finally, two practical cases can be considered:

Concrete plus longitudinal steel plus concrete cover (stirrups and spirals)

$$P_{R0} = F_R(f\,"cA_g + A_s fy) \text{ (First maximum)}$$

Concrete plus longitudinal steel plus transverse steel without concrete cover (only spirals)

$$P_{R0} = F_R (f''c A_c + A_s fy + 2\rho_s fy A_c) \text{ (Second maximum)}$$

In practice it is required that the second maximum will be at least slightly larger than the first maximum, for structural safety, however is not considered as the design capacity.

For this to happen, it is required that the shell contribution would be larger than the contribution of concrete cover.

This will be so if the volumetric reinforcement ratio is not less than:

$$0.45\left(\frac{A_g}{A_c}-1\right)\frac{f'c}{fy} \text{ ; Nor } \quad 0.12\frac{f'c}{fy}$$

Where:

Ag = Gross concrete section

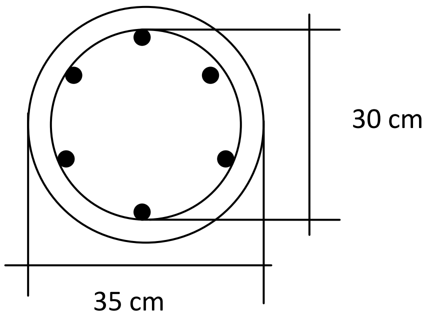
Ac = Core concrete section

As = Area of longitudinal steel

ρs = Volumetric reinforcement ratio

Example 6

Compute first and second maximums for the two s distances:



f´c = 250 kg/cm²

fy = 4200 kg/cm²

As = 6 barras del No. 8

s (1) = 5cm

s (2) =15cm

No. 3 bars for the spirals

Figure 7.3

General procedure for the solution

The First step is to compute the cross concrete section, and the core concrete section.

The Second stepis to compute the contribution to resist axial loads of tranverse reinforcement as a fuction of volumetric reinforcement ratio and the mechanical properties of steel, do this for the two distances "s".

The Third step is compute the first and the second maximum concrete plus longitudinal steel plus concrete cover, in the second maximum we need consider the contribution to resist axial loads of the transverse steel for each distance, computed in the second step.

The Fourth step is to compute the restriction for the volumetric reinforcemente ratio.

Fomulae

Renforcement contribution $P_s = \dfrac{4A_e}{sd}$	First maximum $P_{RO} = F_R(f''_c A_g + A_s f_y)$	Second maximum $P_{RO} = F_R(f''_c A_c + A_s f_y + 2\rho_s f_y A_c)$
Restricition $0.45\left[\dfrac{A_g}{A_c} - 1\right]\dfrac{f'_c}{f_y}$		Restriction $0.12\dfrac{f'_c}{f_y} < P_s$

STEP 1

$$A_g = \frac{\pi\,(35)^2}{4} = 962.11\ cm2$$

$$A_c = \frac{\pi\,(30)^2}{4} = 706.85\ cm2$$

STEP 2

$$P_s = \frac{4A_e}{sd}$$

For “*s*”=5 cm

$$P_{s1} = \frac{4(0.71)}{(5)(30)} = 0.019$$

For “s”= 15 cm

$$P_{s2} = \frac{4(0.71)}{(15)(30)} = 0.0063$$

STEP 3

Let us compute the first maximun :

$$P_{RO} = F_R(f''_c A_g + A_s f_y)$$

$$P_{RO} = (0.8)((170)(962.11) + (30)(4200))$$

$$P_{RO} = 231.64\ ton$$

Second maximum

$$P_{RO} = F_R(f''_c A_c + A_s f_y + 2\rho_s f_y A_c)$$

$$for\ P_{s1} = 0.019$$

$$P_{RO1} = (0.8)((170)(706.85) + (30)(4200) + 2(0.019)(4200)(706.85))$$

$$P_{RO1} = 287.18\ ton$$

$$for\ P_{s2} = 0.0063$$

$$P_{RO2} = (0.8)((170)(706.85) + (30)(4200) + 2(0.0063)(4200)(706.85))$$

$$P_{RO2} = 226.85\ ton$$

STEP 4

So we need that the second maximum will be at least slightly larger tnan the first maximum, for structural safety.

So we need to compute the restriction and compare whit the P_S for the second maximum that is smaller than the first maximum.

$$0.45\left[\frac{A_g}{A_c} - 1\right]\frac{f'_c}{f_y}; \quad or \quad 0.12\frac{f'_c}{f_y} < P_s$$

$$0.45\left[\frac{962.11}{706.85} - 1\right]\frac{250}{4200} = .0096$$

$$0.12\frac{250}{4200} = .0076$$

So the restriction is 0.0096-0.0076 and we need P_s bigger than the resctricion, in this case the P_S who aplly this restriction is 0.019,so we must be used spacing of spiral wire: 5.0 cm.

See Problem V from the annex, page 226

Compression plus bending on rectangular Columns

The structural members that carry only loads in compression are not very common. The presence on bending moments is due to continuity; actually all the members of a structure form a monolithic structure. In addition, bending moments are present due to imperfections in construction generating eccentricities and thus bending moments.

When a member is subjected to combined axial compression P and moment M such in Figure 1a, it is usually convenient to replaced the axial load and moment with and equal load P applied at eccentricity e = M/P as in Figure 1b. The two loading are statically equivalent.

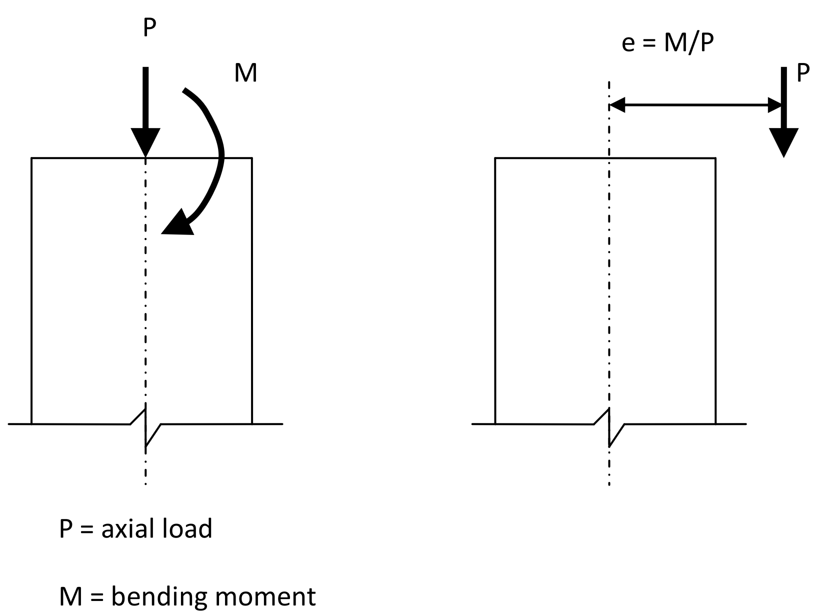


Figure 8.1 Compression plus bending

All columns can be classified in terms of the equivalent eccentricity.

Those having relatively small e, are generally characterized by compression over the entire concrete section, and if over loaded will fail by crushing of the concrete accompanied by yielding of the steel in compression on the more heavily loaded side.

Columns with large eccentricity are subject to tension over at least a part of the section, and if over loaded may fail due to tensile yielding of the steel on the farthest side from the load P.

For large eccentricities, failure is initiated by yielding of the tension steel As. Hence, for this case, fs = fy. When the concrete reaches its ultimate strain, εc, the compression steel may or may not have yielded; this must be determined by compatibility of strains.

For small eccentricities the concrete will reach its limit strain, εc, before the tension steel starts yielding; in fact, the bars on the side of the column farther from the load may be in compression, not tension. This must be determined by a compatibility of strains, too.

An approach providing the basis for practical design is to construct a strength interaction diagram defining the failure load and failure moment for a given column for the full range of eccentricities from cero to infinity. For any eccentricity, there is a unique pair of values of Pn and Mn that will produce the state of incipient failure. That pair can be plotted as a point on a graph relating Pn and Mn (Figure 8.2)

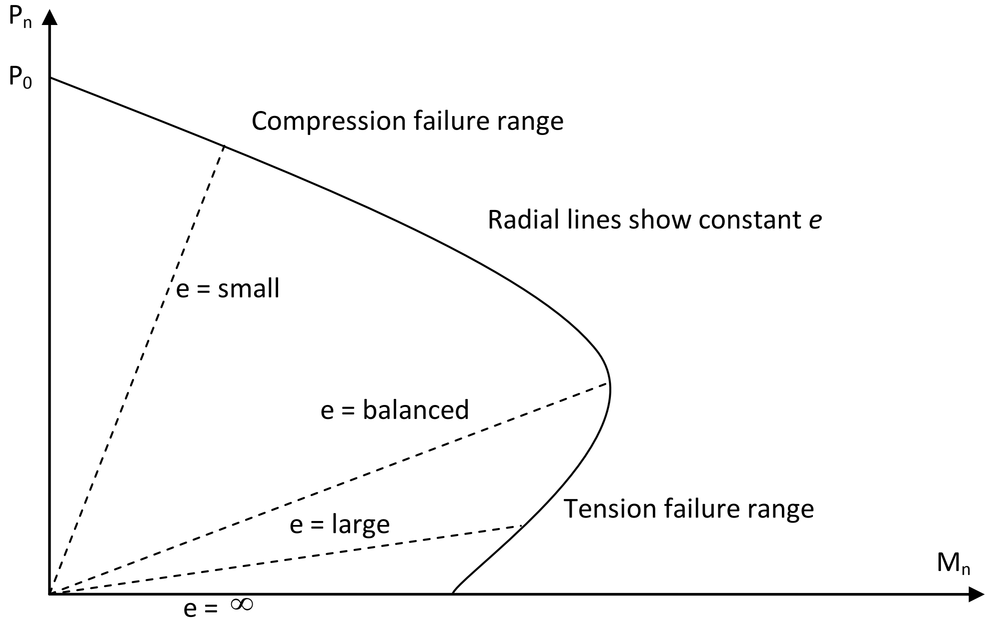


Figure 8.2 Column interaction diagram

Balanced failure

Given an eccentricity, eb, an axial load, Pb, and a bending moment, Mb. The balanced failure is reached when simultaneously the concrete and steel reach their limit strains, εc and εs, respectively.

This point in the interaction diagram is the dividing point between compression failure and tension failure.

It is to be noted that, in contrast to beam design, one cannot restrict column designs such that yielding failure rather than crushing failure would always be the result of overloading.

Different stages for a column loading are shown in Figure 8.3.

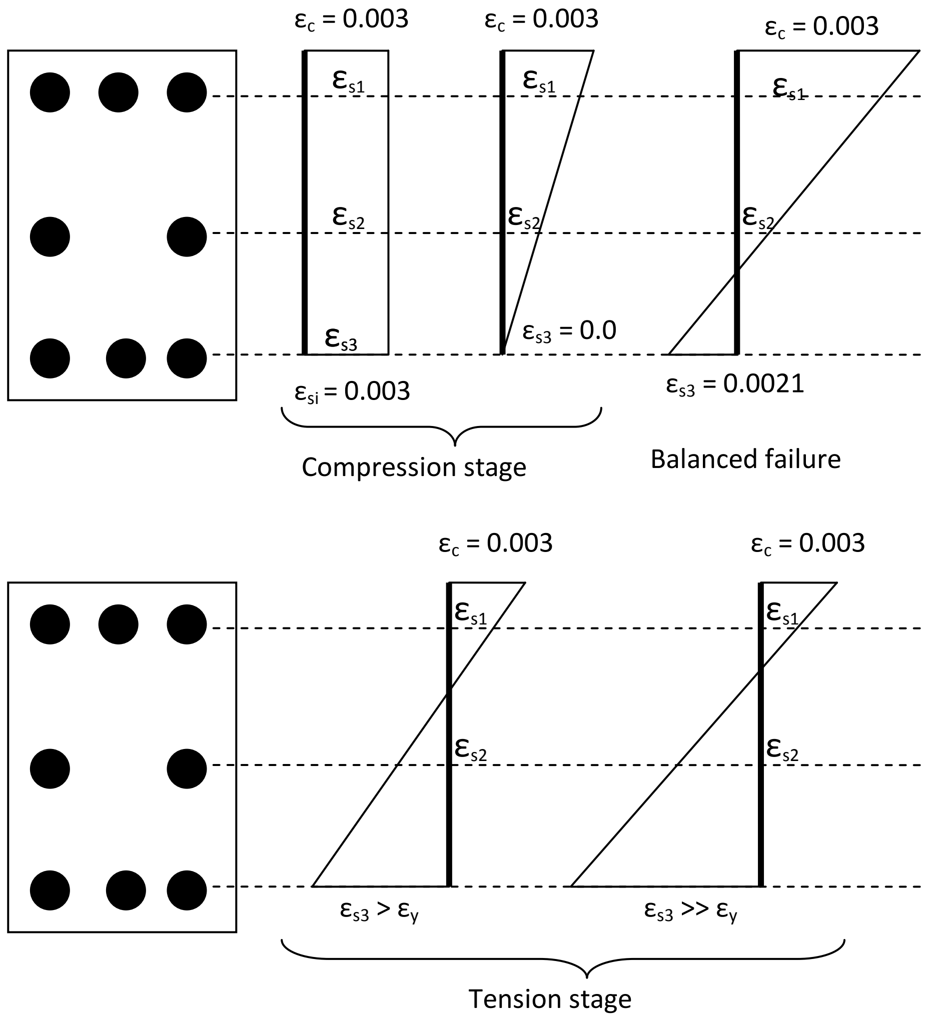


Figure 8.3 Stages for a column loading in compression plus bending

Example 7

Construct the strength interaction diagram for the square column.

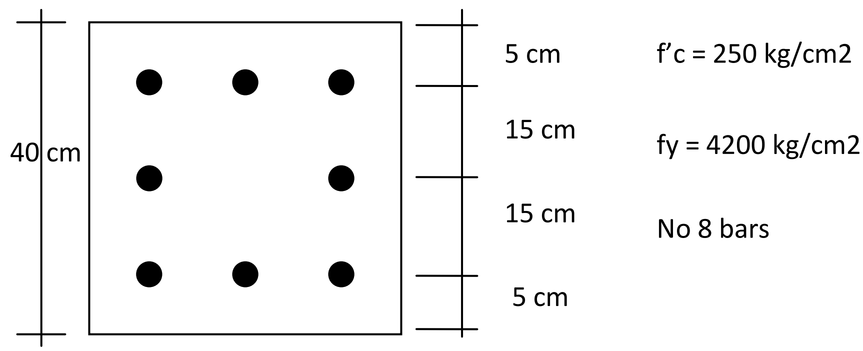


Figure 8.4

General procedure for the solution

First step, Compute the forces in concrete and steel by computing the unit strains in the different beds of steel.

Second step, compute the axial total force and then the resistance moment of the section under the considered distributions of unit deformations.

Formulae

Force	Distance	Total forces	compression
$p = A_g f''_c + A_s f_y$	$E_{s1} = \dfrac{(c-5)(E_c)}{c}$	$p = c_{s1} + c_c + c_{s2}$	$C = (0.85)(c)(b)\big(f''_c\big)$
Compression $c = \dfrac{E_c\, d}{E_c + E_{S3}}$	$c_{s1} = A_{s1} f_y$		

STEP 1

Point one

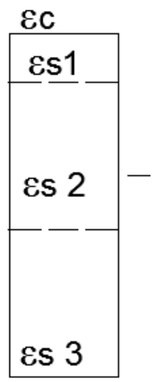


Figure 8.6

$$p = A_g f''_c + A_s f_y$$

$$p = (1600)(170) + (40)(4200)$$

$$p = 440\ ton$$

STEP 2

Point two

Let us compute E_s :

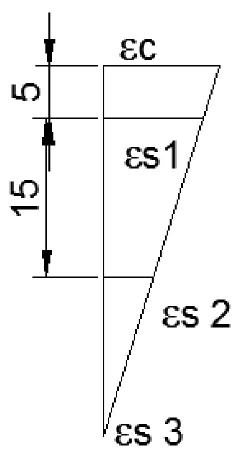


Figure 8.7

$$E_c = 0.003$$

$$c = 35$$

For E_{s1} we take the distance for E_c to E_{s1} that shows in the figure, and check if is yielding or not to consider f_s .

$$E_{s1} = \frac{(c-5)(E_c)}{c}$$

$$E_{s1} = \frac{(35-5)(0.003)}{35} = 0.0027$$

$E_{s1} > .0021$ It's yielding so, $f_s = f_y$

For E_{s2} we take the distance for E_c to E_{s2} that shows in the figure, and check if is yielding or not to consider f_s .

$$E_{s2} = \frac{(35-20)(0.003)}{35} = 0.0012$$

$$E_{s2} < .0021 \text{ It's not yielding so, } f_s < f_y$$

$$f_s = E_s \in_s$$

$$f_s = (2x10^6)(0.0012) = 2{,}400\ kg/cm2$$

STEP 3

Let us compute the forces

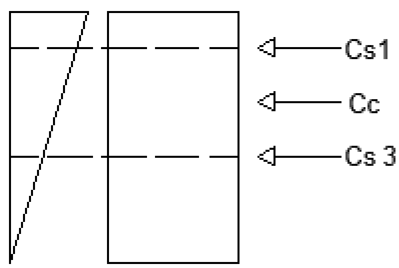


Figure 8.8

$$p = c_{s1} + c_c + c_{s2}$$

c_c Is for the compression for the concrete,

$$C = (0.85)(c)(b)\left(f''_c\right)$$

$$c_c = (0.85)(35)40(170) = 202.3\ ton$$

c_{s1} Is for the force that provide the steel with the area for the steel in this case is #3 bars, so A_{s1} it's equal to 15 cm^2

$$c_{s1} = A_{s1}f_y$$

$$c_{s1} = (15)(4200) = 63\ ton$$

c_{s2} Is for the force that provide the steel with the area for the steel in this case is #2 bars, so 2 it's equal to 10 cm^2

$$c_{s2} = A_{s2}f_y$$

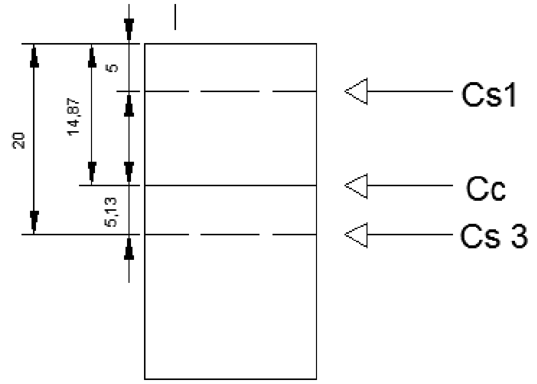
$$c_{s2} = (10)(2400) = 24\ ton$$

So, the total force is the addition for the three forces.

$$p = 63 + 202.3 + 24 = 289.3\ ton$$

STEP 4

Let us compute M



$$Arm(l)Cc\ is = \frac{0.85 * 35}{2} = 14.87$$

$$M_1 = (63)(.15) = 9.45\ ton - m$$

$$M_2 = (202.3)(0.0513) = 10.37\ ton - m$$

$$M_3 = (24)(0) = 0\ ton - m$$

$$M_{total} = 9.45 + 10.37 = 19.82\ ton - m$$

figure 8.9

Point 3

At this point we need to calculate the new "c" and repeat the fourth steps that we did in the point two, E_{S3} it´s tacking from the text and the diagrams for the balanced failure.

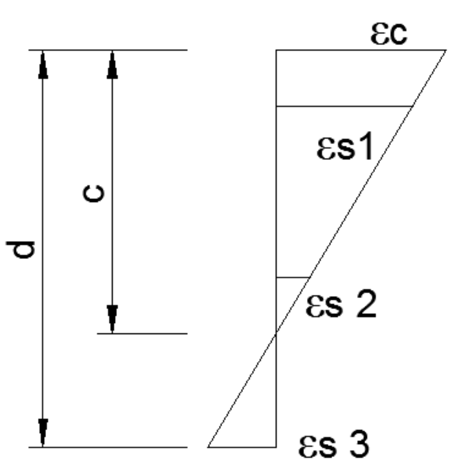


Figure 8.10

$$c = \frac{E_c\, d}{E_c + E_{S3}}$$

$$c = \frac{(0.003)(35)}{(0.003) + (0.0021)}$$

$$c = 20.58\ cm$$

For E_{s1} we take the distance "c" that we compute, and distance for E_c toE_{s1}, and check if is yielding or not to consider f_s .

$$E_{s1} = \frac{(c - 5)(E_c)}{c}$$

$$E_{s1} = \frac{(20.58 - 5)(0.003)}{20.58} = 0.0027$$

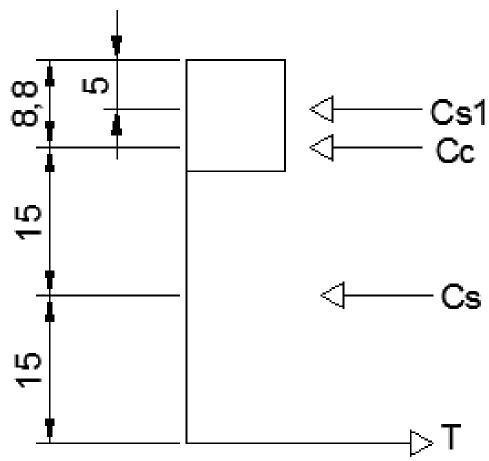
$$E_{s2} = \frac{(20.58 - 20)(0.003)}{20.58} = 0.0000845$$

$E_{s2} < .0021$ It's not yielding so, $f_s < f_y$

$$f_s = E_s E_{s2}$$
$$f_s = (2x10^6)(0.0000845) = 169\ kg/cm2$$

$E_{s1} > .0021$ It's yielding so, $f_s = f_y$
For E_{s2} we take de distance "c" that we compute, and distance for E_c toE_{s2}, and check if is yielding or not to consider f_s .

Let us compute the forces



$$c_c = (0.85)(20.58)(40)(170) = 118.95\ ton$$

$$c_{s1} = (15)(4200) = 63\ ton$$

$$c_{s2} = (10)(168) = 1.68\ ton$$

$$T = (4200)(15) = 63\ ton$$

$$p = 63 + 1.68 - 63 + 118.95 = 120.63\ ton$$

Figure 8.11

Let us compute the moments

$$Arm(l)\ Cc\ is = \frac{(.856)(20.58)}{2} = 8.8$$

$$M_1 = (63)(.15) = 9.45\ ton - m$$

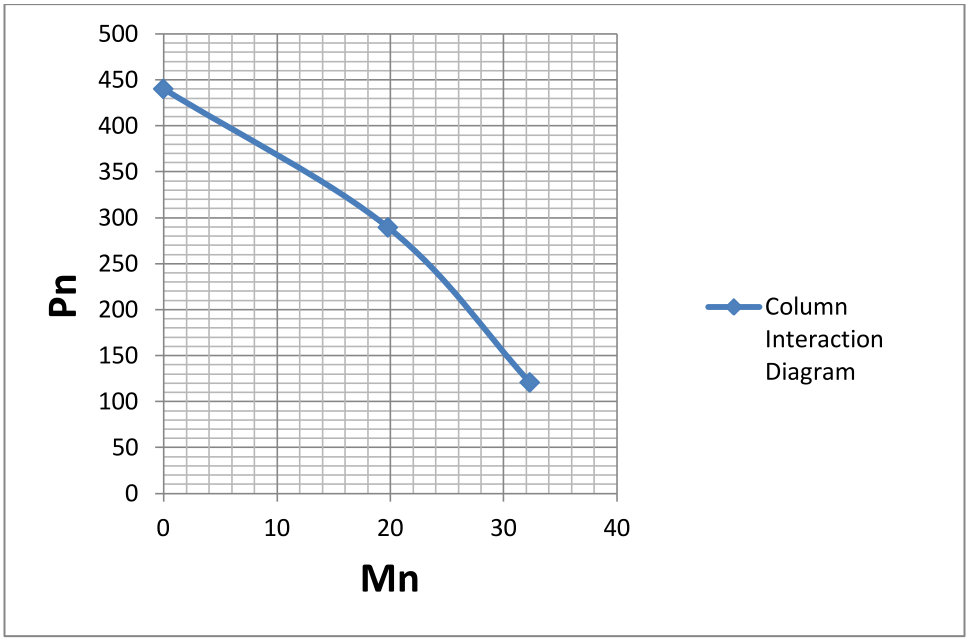
$$M_2 = (1.68)(0) = 0\ ton - m$$

$$M_3 = (63)(.15) = 9.45\ ton - m$$

$$M_4 = (118.95)(.113) = 13.44\ ton - m$$

$$M_{total} = 9.45 + 9.45 + 13.44 = 32.34\ ton - m$$

Column interaction diagram.

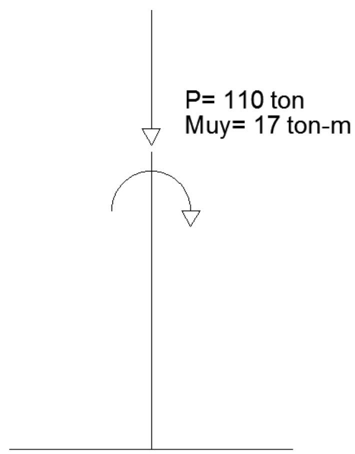


See Problem VI from the annex, page 227

Example 8

From a structural analysis of a structure the following forces are found to be acting on a column. Pu = 110 ton and Muy = 17 ton-m

Propose a reinforced concrete section to resist the forces.



General Procedure

The First step is to check de minimum moment

$$M_{min} = e_{min}p$$

The Second step is to propose a concrete section and the steel reinforcement using the design charts (Appendix C- Gonzalez Cuevas)

Formulae:

Minimum moment	Factor	Steel area
$M_{min} = e_{min}p$	$R = \frac{M_u}{F_R b h^2 f'_c}$	$A_s = \rho b h$
Factor $k = \frac{p_u}{F_R b h f'_c}$	$\rho = \frac{A_s}{bh}$	

STEP 1

We need to check the minimum moment.

$$M_{min} = e_{min}p$$

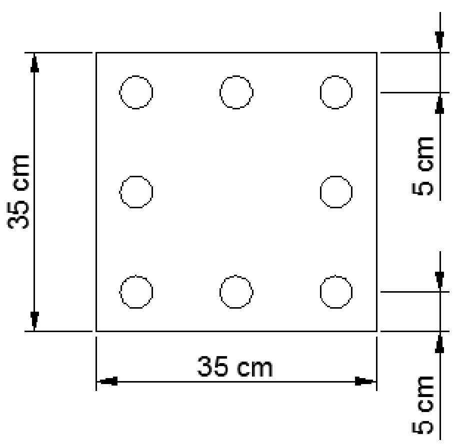
$$e_{min} = 0.02m$$

$$M_{min} = (0.02)(110) = 2.2\ ton - m$$

As the minimum moment is smaller than the moment that is acting on the column, we take the later as the design moment (17 ton-m).

STEP 2

First propose section



$$f'_c = 250\ kg/cm2$$

$$f_y = 4200\ kg/cm$$

$$\frac{d}{h} = \frac{30}{35} = 0.85$$

Figure 8.12

$$k = \frac{110000}{(0.8)(35)(35)(250)} = 0.448$$

$$R = \frac{1700000}{(0.9)(35)(35^2)(250)} = 0.17$$

With, k and R we can find from the chart “q”. So, with k=0.448 and R=0.17 we found, q=0.96

$$\rho = q\,\frac{f''_c}{fy}$$

$$\rho = 0.96\,\frac{170}{4200} = .03885$$

$$A_s = \rho bh$$

$$A_s = (.03885)(35)(35) = 47.6\ cm2$$

$$A_S \approx 50\ cm2 = 10\ bars\ \#8$$

As we see that we have a lot of bars, we will propose a large concrete section to reduce amount of steel.

So, we propose h=40, with the same recover.

$$\frac{d}{h} = \frac{35}{40} = 0.9$$

$$k = \frac{110000}{(0.8)(40)(40)(250)} = 0.34$$

$$R = \frac{1700000}{(0.9)(40)(40^2)(250)} = 0.118$$

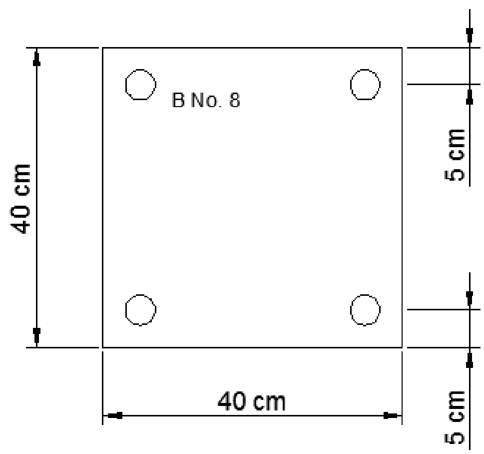
With, k and R we can find from the chart “q”. So, with k=0.34 and R=0.118 we found, q=0.28

$$\rho = 0.28\ \frac{170}{4200} = .0113$$

$$A_s = (.0113)(40)(40) = 17.6\ cm2$$

$$A_S \approx 18\ cm2 = 4\ bars\ \#8$$

So the final reinforced section is:



$$f'_c = 250\ kg/cm2$$

$$f_y = 4200\ kg/cm2$$

Figure 8.13

See Problem VII from the annex, page 228

Compression plus bi-axial bending

The methodology studied in last session can be applied when compression plus bending moments are acting in one principal direction.

However, almost always, it is compression plus bending moments acting in two principal directions. Think about a corner column, as an example.

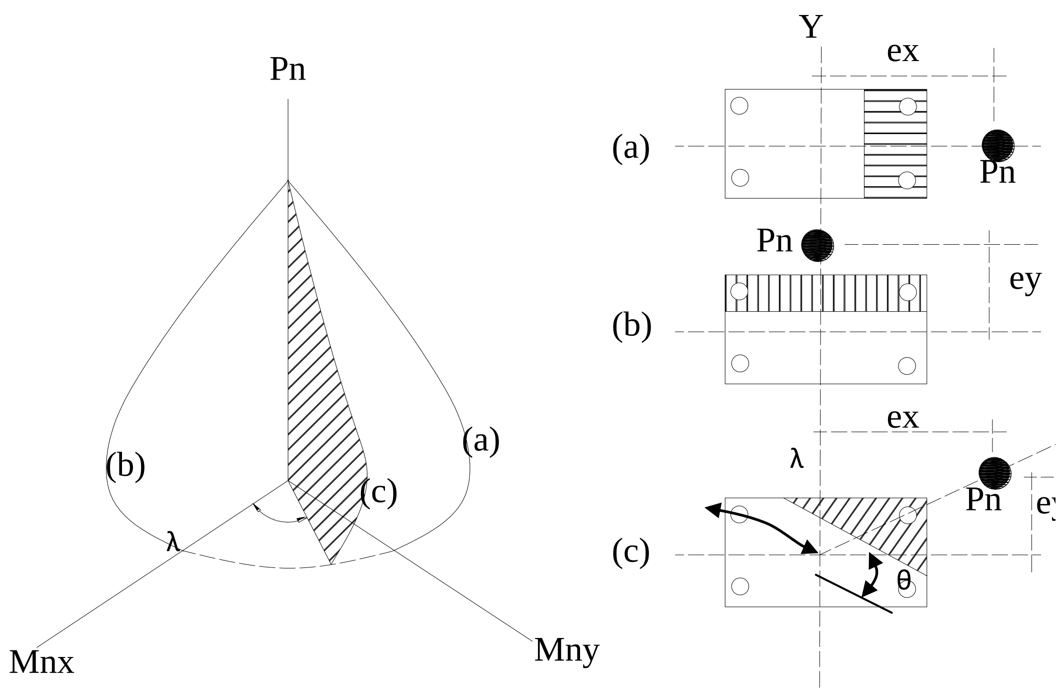


Figure 9.1 Column interaction diagram for axial load plus bending moments in two directions.

The angle at which is acting the load can be computed by the following expression:

$$\lambda = \arctan\frac{e_x}{e_y} = \arctan\frac{M_{ny}}{M_{nx}}$$

In order to construct an interaction diagram, like that on Figure1, it is done in similar fashion than for axial load plus bending moment in one direction.

In practice, the axial load P and both bending moments (Mx y My) are known from the structural analysis, so it is possible to compute the following unknowns: λ, ex y ey.

Reciprocal load method

$$\frac{1}{P_n}=\frac{1}{P_{nx0}}+\frac{1}{P_{ny0}}-\frac{1}{P_0}$$

Where:

Pn = Approximate value of nominal load in biaxial bending with eccentricities ex and ey.
Pnx0 = Nominal load when only eccentricity on y is present (ex = 0)
Pny0 = Nominal load when only eccentricity on x is present (ey = 0)
P0 = Nominal load for concentrically loaded column

Note that the last equation reduces the problem to a combination of three simple solutions: two in uniaxial bending and one in pure compression.

In a typical design situation, given the size and reinforcement of the trial column and the load eccentricities (ex and ey), one finds by computation or from design charts the nominal loads Pnx0 and Pny0 for uniaxial bending around the X and Y axes, respectively, and the nominal load P0 for concentric loading. Then 1/Pn is computed and from that, Pn is calculated.

Where the equation comes from?

Note that the interaction diagram can be plotted as a function of Pn and the eccentricities ex y ey (Figure 9.2a). Now the same interaction diagram is plotted as a function of 1/ Pn and the eccentricities ex y ey (Figure 9.2b).

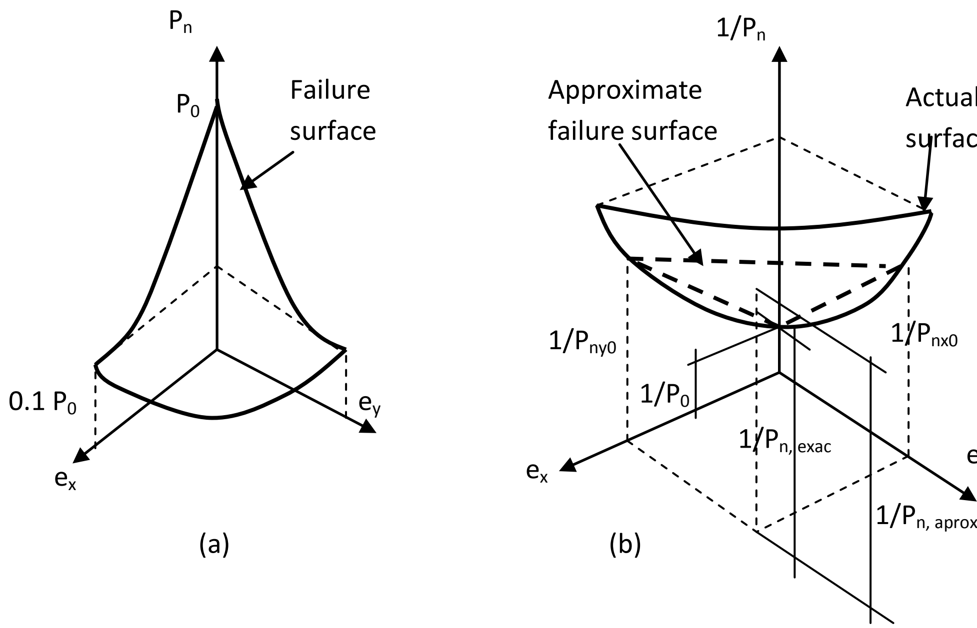


Figure 9.2 Interaction surfaces for the reciprocal load method

The reciprocal load equation has been found to be acceptably accurate for design purposes provided Pn > 0.1 P0. It is not reliable when biaxial bending is prevalent and accompanied by and axial force smaller than P0/10.

In this case of such strongly prevalent bending, failure is initiated by yielding of the steel in tension.

This situation corresponds to the lowest tenth of the interaction diagram.

In this range, it is conservative and accurate enough to neglect the axial force entirely and to calculate the section for biaxial bending only.

Minimum eccentricity NTC-2004

The design eccentricity must not be less than 0.05h ≥ 20 mm, where h is the section dimension in which bending is considered.

For values of Pn < 0.1 P0 the design expression will be:

$$\frac{M_{ux}}{M_{RX}} + \frac{M_{uy}}{M_{Ry}} \leq 1.0$$

Where:

Mux y Muy are the design moments around X and Y
MRx y MRy are the resistant moments around X and

Example 9

From a structural analysis of a structure the following forces are found to be acting on a column. Pu = 110 ton, Muy = 17 ton-m and Mux = 8 ton-m.

Check the adequacy of the trial design using the reciprocal method.

Trial column:

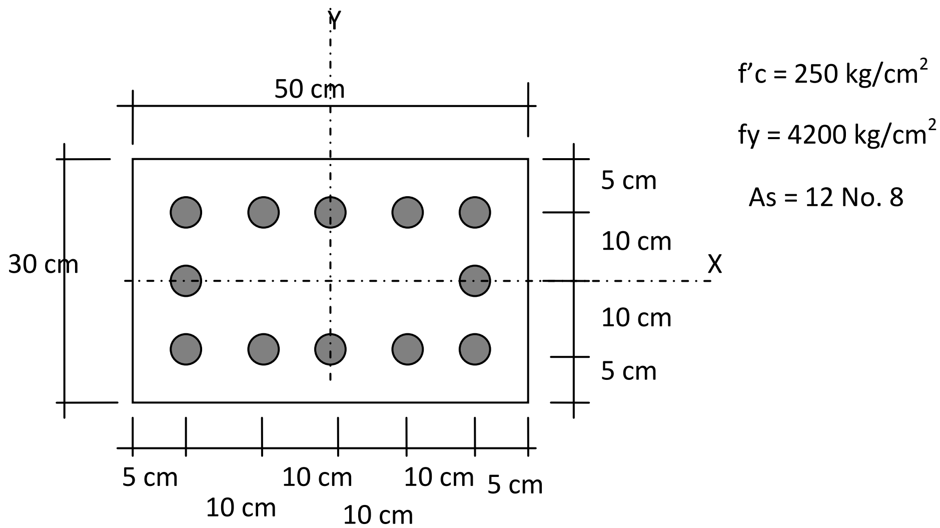


Figure 9.3

General procedure

The First step is to compute the Minimum moments

The Second step is to Proposed a reinforced concrete section (dimensions and quantity of steel).

The Third step is to Compute Pox, Po and to compute Pn

Formulae:

$\frac{d}{h}$	$R = \frac{M_u}{F_R b h^2 f'_c}$	$\rho = \frac{A_S}{bh}$	$q = \rho \ \frac{fy}{f''_c}$	$P_{nyo} = kbhf'_c$
	$P_o = A_S fy + A_g f''_c$		$P_r = F_R P_o$	$\frac{1}{P_n} = \frac{1}{P_{nxo}} + \frac{1}{P_{nyo}} - \frac{1}{P_o}$

STEP 1

Minimum moment

$$Mmin = \varrho_{min} \ x \ P$$

In X direction

$$Mminy = \varrho_{minx} \ x \ P$$

$$\varrho_{minx} = 0.05 \ (50)$$

$$\varrho_{minx} = 2.5 > 2.0$$

$$Mmin = (0.025)x(110)$$

$$Mmin = 2.75 \ ton - m$$

$$Muy > Mminy$$

So, the design moment is Muy

In Y direction

$$Mminx = \varrho_{miny} \ x \ P$$

$$\varrho_{minx} = 0.05 \ (30)$$

$$\varrho_{minx} = 1.5 < 2.0$$

$$Mmin = (0.02)x(110)$$

$$Mmin = 2.2\ ton - m$$

$$Mux > Mminx$$

So, the design moment is Mux

For “y” and Second step for “y”

$$\text{For } Muy = 17\ ton - m$$

$$\frac{d}{h} = \frac{45}{50} = 0.9$$

$$R = \frac{M_u}{F_R b h^2 f'_c}$$

$$R = \frac{1700000}{(0.9)(30)(50^2)(250)} = 0.10$$

STEP 2

$$\rho = \frac{A_S}{bh}$$

$$\rho = \frac{60}{(30)(50)} = 0.04$$

$$q = \rho\ \frac{fy}{f''_c}$$

$$q = 0.04\ \frac{4200}{170} = 0.98$$

So, we need to check “k” in the tables (R,q)

P_{nyo}

$$Muy = 17\ ton$$

$$Pu = 110\ ton$$

$$e = \frac{17}{110} = 0.15$$

$$\frac{e}{h} = \frac{15}{50} = 0.3$$

In order to know the proper chart we need to compute $\frac{d}{h}$

$$\frac{d}{h} = \frac{45}{50} = 0.9$$

$$K = 0.55$$

$$P_{nyo} = kbhf'_c$$

$$P_{nyo} = (0.55)(30)(50)(250) = 206250\ kg$$

$$P_{nyo} = 206.25\ ton$$

First step for “X” and Second step for “X”

P_{nxo}

For $Mux = 8\ ton - m$

$$Pu = 110\ ton$$

$$e = \frac{8}{110} = 0.072$$

$$\frac{e}{h} = \frac{7.2}{30} = 0.2424$$

In order to know the proper chart we need to compute $\frac{d}{h}$

$$\frac{d}{h} = \frac{25}{30} = 0.83$$

$$K = 0.7$$

$$P_{nxo} = kbhf'_c$$

$$P_{nxo} = (0.7)(50)(30)(250) = 262500\ kg$$

$$P_{nxo} = 262.5\ ton$$

$$\rho = \frac{60}{(30)(50)} = 0.04$$

$$q = 0.04\ \frac{4200}{170} = 0.98$$

STEP 3

Let us compute Po

$$P_o = A_S fy + \ A_g f''_c$$

$$P_o = (60)(4200) + (30 * 50)(170) = 507000\ kg$$

$$P_o = 507\ ton$$

Computing Pn

$$\frac{1}{P_n} = \frac{1}{P_{nxo}} + \frac{1}{P_{nyo}} - \frac{1}{P_o}$$

$$\frac{1}{P_n} = \frac{1}{262.5} + \frac{1}{206.25} - \frac{1}{507} = 6.68x10^{-03}$$

$$P_n = 149.59\ ton$$

$$P_r = F_R P_o$$

$$P_r = (0.8)(149.59) = 120\ ton$$

See Problem VIII from the annex, page 229

Slender Columns

The material presented so far pertained to concentrically or eccentrically loaded short columns, for which the strength is governed entirely by the strength of the materials and the geometry of the section.

Most columns in present-day practice fall in this category.

However, with the increasing use of high-strength materials and improved methods of dimensioning members, it is now possible to design much smaller cross section members than in the past. This clearly makes for more slender members.

A column is said to be slender if its cross section dimensions are small compared with its length.

The degree of slenderness is generally expressed in terms of the slenderness ratio l/r, where l is te supported length of the member and r is the radious of gyration of the cross section (I/A)1/2.

For square or circular members the value of r is the same about either axis; for other shapes r is the smallest about the minor principal axis.

It is known that a member of great slenderness will collapse under a smaller compression load tan s stocky member with the same cross section (figure 10.1).

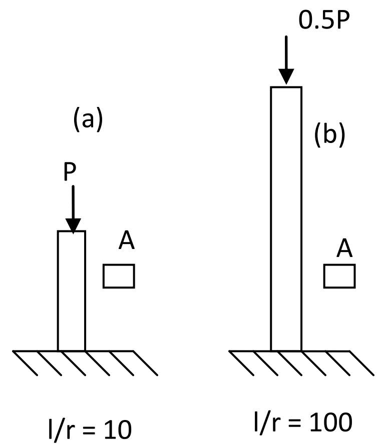


Figure 10.1 Short and slender columns

The collapse of column b will be cause by buckling, by sudden lateral displacement of the member between its ends.

Associated with these lateral displacements are secondary moments that add to the primary moments and that may become very large for slender columns, leading to failure.

A practical definition of a slender column is one for which there is a significant reduction in axial load capacity because of these secondary moments.

Effect of slenderness on the carrying capacity of a column

A slender reinforced concrete column reaches the limit of its strength when the combination of M and P at the most highly stressed causes the section to fail. That means that M and P approaches to and becomes equal to Pn and Mn. This can be seen in figure 10.2

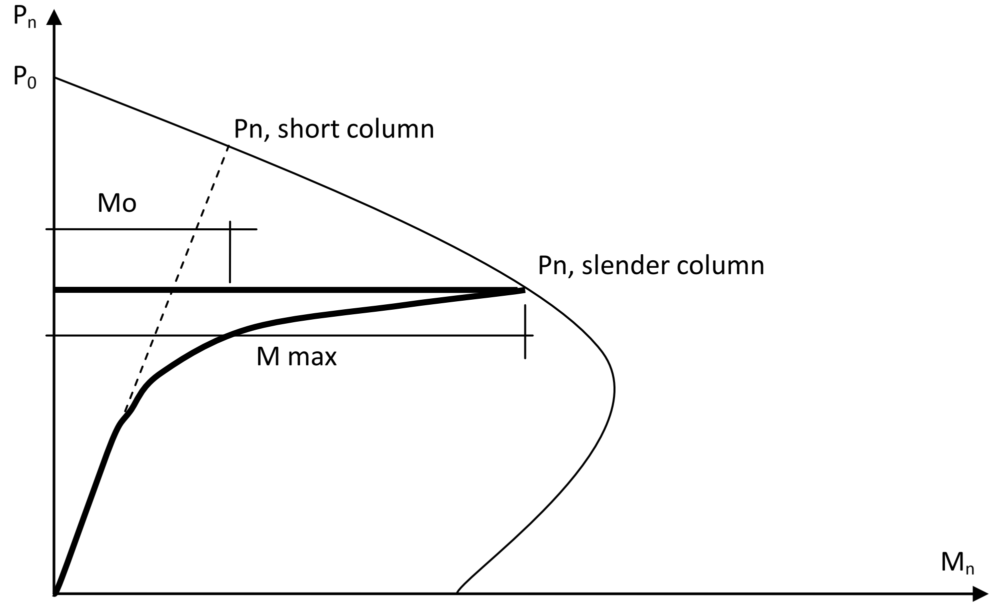


Figure 10.2 Interaction diagram for a slender column.

Concentrically loaded columns

The basic information on the behavior of straight, concentrically loaded slender columns is generalized by Euler equation; it states that such a member will fail by buckling at the critical load Pc.

$$P_c = \frac{\pi^2 E_t I}{(kl)^2}$$

It is seen that the buckling load decreases rapidly with the increasing slenderness ratio (kl/r). This can be seen in Figure 10.3.

$$Pn = A_s fy + A_g f"c$$ Short column

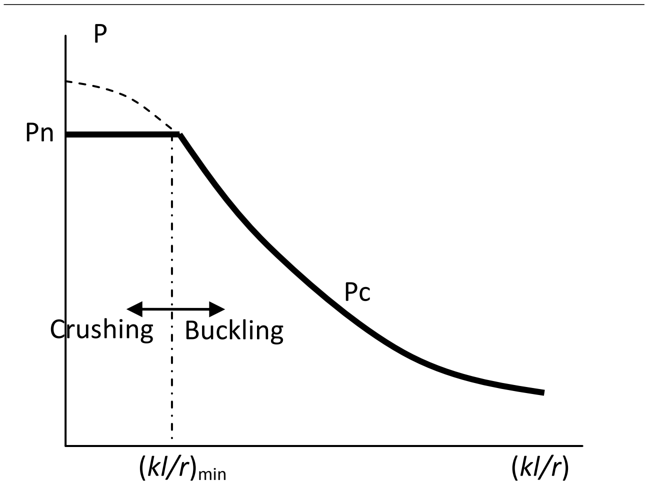


Figure 10.3 Column curve

Figure 10.4 presents graphically the effective length and buckling mode of columns.

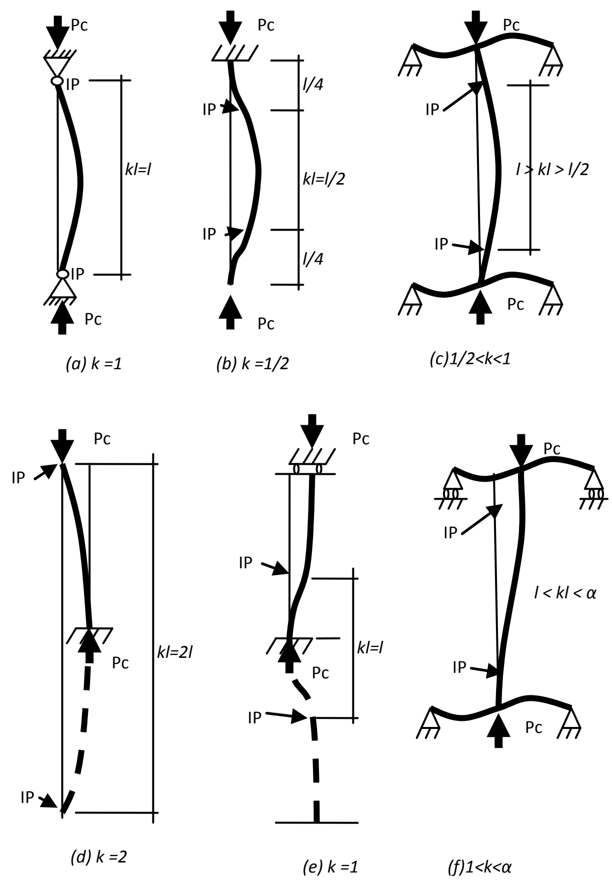


Figure 10.4 Buckling and effective length of axially loaded columns.

Figure 10.5 show a rigid frame buckling: a) laterally braced, b) laterally unbraced

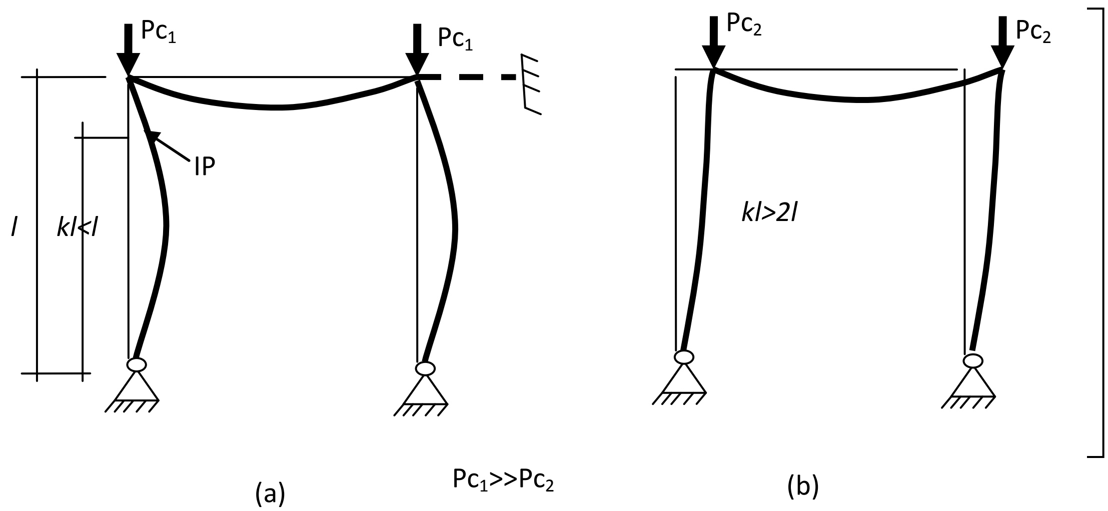


Figure 10. 5. Buckling on rigid frames

Figure 10.5 is an illustration of the general fact that compression on member free to buckle in a sideway mode are always considerably weaker than when braced against sideway.

In summary, the following can be noted:

The strength of concentrically loaded columns decreases with increasing slenderness ratio.

In columns that are braced against sideway or that are parts of frames braced against sideway, the effective length kl, falls between l/2l and l, depending on the degree of end restraints.

The effective lengths of columns that are not braced against sideway or that are parts of frames not so braced are always larger than l, the more so the smaller the end restraint. In consequence, the buckling load of a frame not braced against sideway is always substantially smaller than that of the same frame when braced.

Compression plus bending

Most reinforced concrete compression members are also subject to simultaneous flexure, cause by transverse loads or by end moments owing to continuity.

The behavior of members subject to such combined loading also depends greatly on their slenderness.

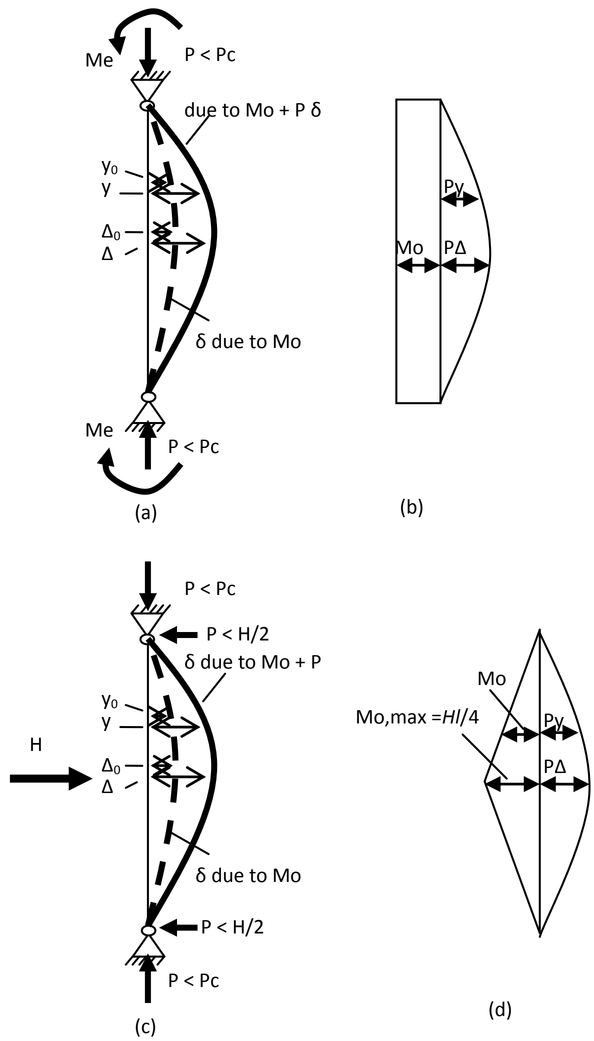


*Figure 10.6 Moments in slender members with compression plus bending, bent in single***.**

From Figure 10.6 the total moment is:

$$M = M_0 + Py$$

This is one illustration of the so-called P-Δ effect.

A similar situation is shown in Figure 10.6c, where bending is caused by a transverse load H.

The deflections y of elastic columns can be calculated from yo using the following expression:

$$y = y_0 \frac{1}{1 - \frac{P}{P_c}}$$

If Δ is the deflection at the point of maximum moment M max, the amplified moment can be calculated as following:

$$M_{\max} = M_0 + P\Delta = M_0 + P\Delta_0 \frac{1}{1 - P/P_c}$$

Last equation can be written:

$$M_{\max} = M_0 \frac{1 + \psi P/P_c}{1 - P/P_c}$$

Considering that ψ depends on the type of loading and varies between about + 0.2 in most practical cases. Because P/Pc is always smaller than 1, the second term in the numerator is small enough to be neglected, so the last equation is simplified:

Where:

$$\frac{1}{1 - P/P_c}$$

Is known as the moment magnification factor, which reflects the amount by which the moment Mo is magnified by te effect P-Δ.

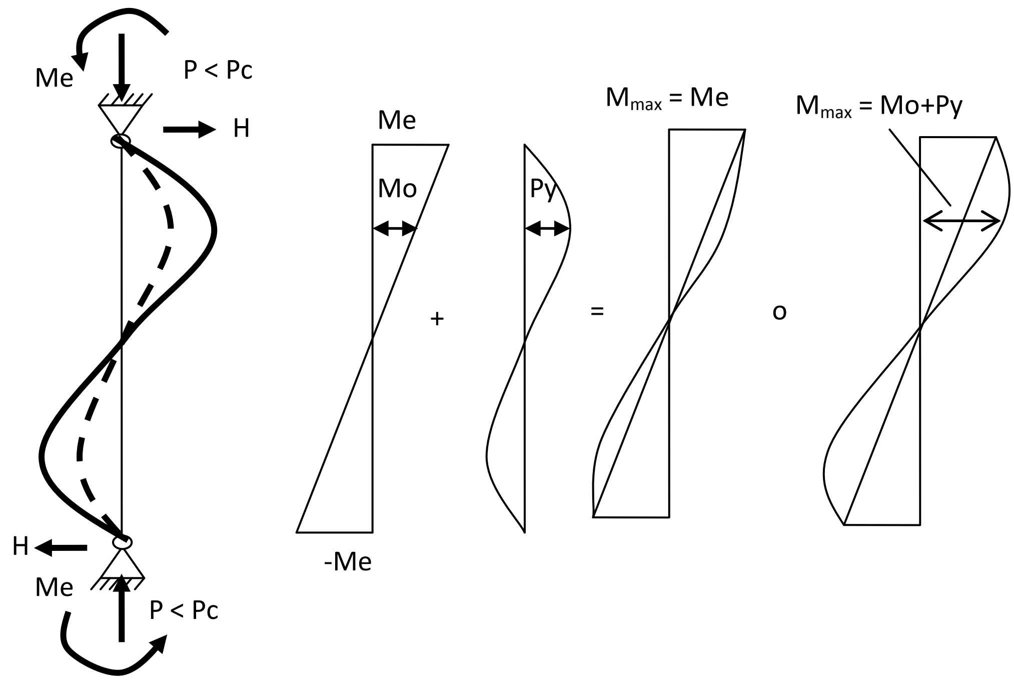


Figure 10.7 Deflections and moments in slender members with compression plus bending bent in double curvature.

In Figure 10.6 it can be seen how for a slender element bent in single curvature the magnification factor is large while in Figure 10.8 it can be seen that the magnification factor for a slender element bent in double curvature, it is small or null, so little or no magnification is presented.

This can be taken into account by a modification of the amplification factor:

$$M_{\max} = M_0 \frac{C_m}{1 - P / P_c}$$

Where:

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4$$

In this expression M1 is the numerically smaller moment and M2 is the numerically larger moment. By definition Mo = M2.

The fraction M1/M2 is defined as positive if the end moments produce single curvature and negative if they produce double curvature.

It is seen that when M1 = M2 (numerically) Cm = 1.

Last equation applied only to members braced against sideway (Figure 10.8).

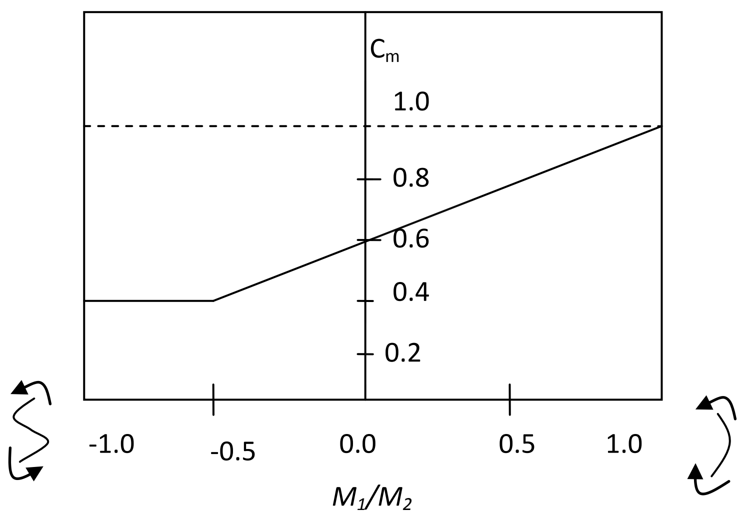


Figure 10.8 Values of Cm for slender columns in nonsway frames.

For sway frames Come will be equal to the unit. This can be seen in Figure 10.9

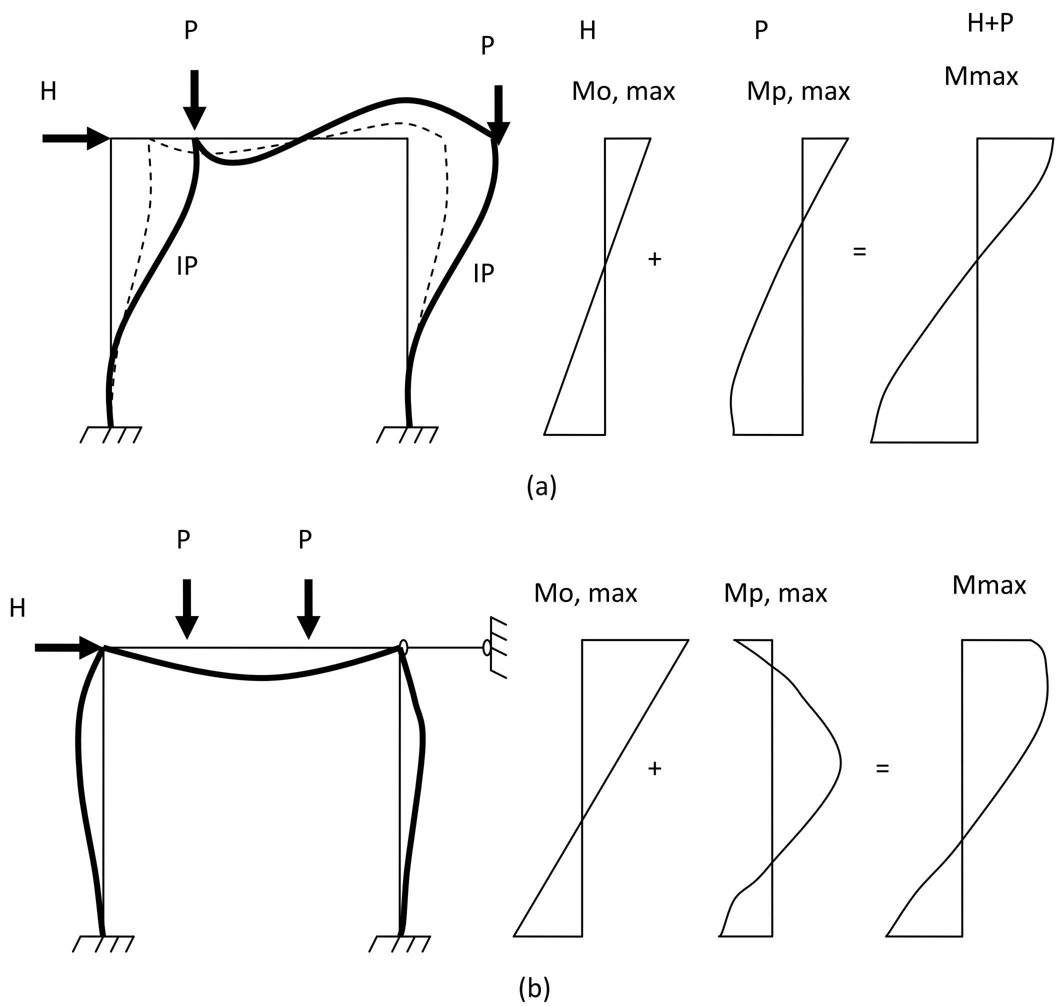


Figure 10.9 (a) Laterally unbraced (sway) frame and (b) Laterally braced (nonsway) frame.

Summary

In flexural members, the presence of axial compression causes additional deflections and additional moments Py. Others things been equal, the additional moments increase with increasing slenderness ratio.

In member braced against sideway and bent in single curvature, the maxima of both types of moments, Mo y Py, occur at the same or at nearby locations and are fully additive, this leads to a large magnification. If the moments Mo result in double curvature very little or no moment magnification occurs.

In members in frames not braced against sideway, the maximum moments of both kinds, Mo and Py, almost always occur at the same locations and are fully additive. Here too, others things been equal, the additional moments increase with increasing slenderness ratio .

Criteria for neglecting of slenderness effects NTC-2004

The procedure of designing slender columns is inevitably lengthy, particularly because it involves a trial and error process.

On the other hand, in most cases columns fall in the short type, so it is necessary to know when slenderness effect can be neglected.

In actual structures, a frame is seldom either completely braced or completely unbraced. It is necessary, therefore, to determine in advance if bracing provided by shear walls, elevator walls, stairwells, or other elements is adequate to restrain the frame against sway effects.

In element in non sway frames the slenderness effects can be neglected when:

$$\frac{kl_u}{r} \leq 34 - 12\frac{M_1}{M_2}$$

Where:

$$34 - 12\frac{M_1}{M_2}$$

must not be larger than 40

The fraction M1/M2 is defined as positive if the end moments produce single curvature and negative if they produce double curvature if M1 = M2 = 0 the fraction M1/M2 will be taken as 1.

In elements in a sway frames the slenderness effects can be neglected when:

$$\frac{kl_u}{r} < 22 \quad \text{(ACI)}$$

In elements in a sway frames the slenderness effects cannot be neglected NTC DF-2004.

Second order analysis

When H'/r is larger than 100, a second order analysis must be carried out in both sway and nonsway frames.

A second order analysis takes into account:

- Material cracking
- Deformation
- No linear behavior of materials
- Load application
- Volumetric changes in elements, and many others.

Today most structural programs can compute second order analysis.

Analysis procedure

Figure 10.10 shows a diagram to carry out a slenderness analysis NTC-2004

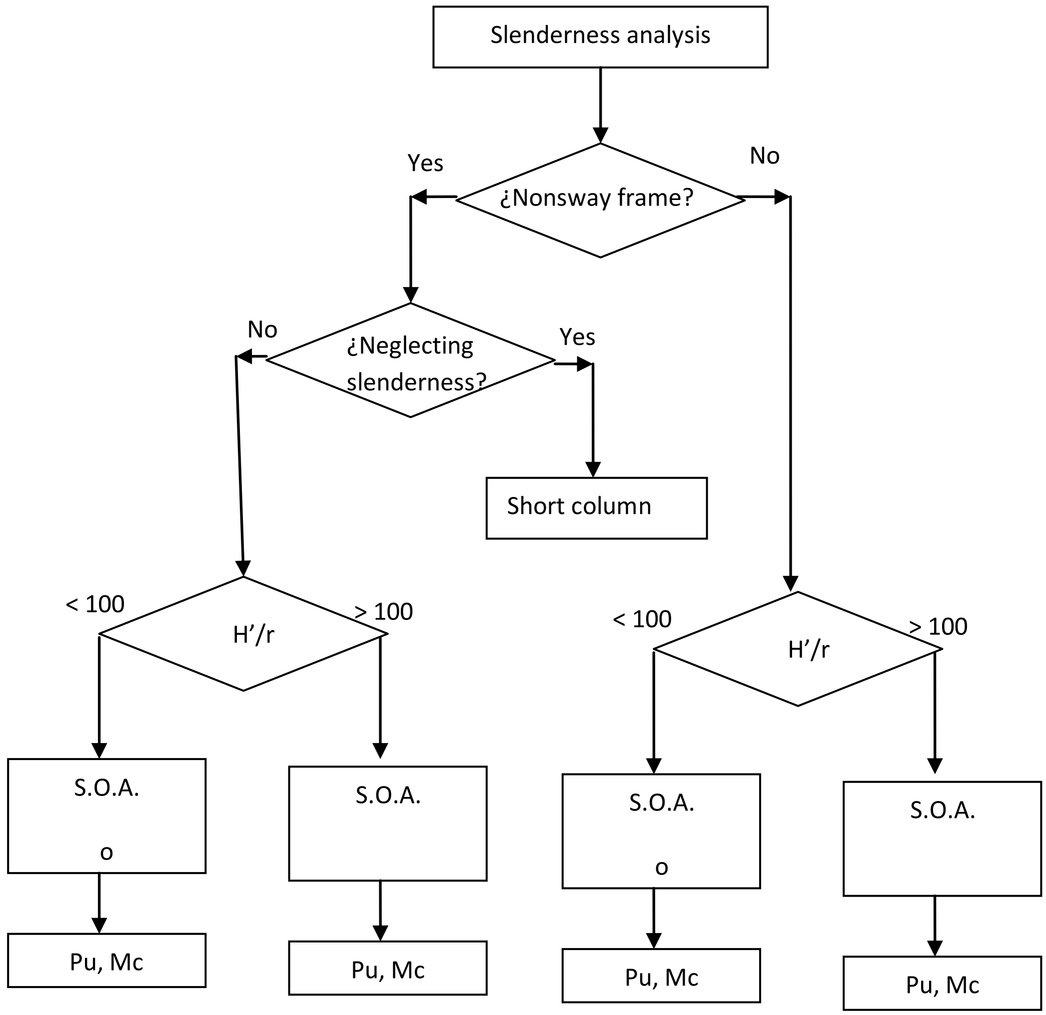


Figura 10.10 Analysis procedure

S.O.A = Second Order Analysis

M.M.M = Moment Magnifier Method

Moment magnifier method for nonsway frames NTC-DF 2004.

The elements will be design with a magnified moment Mc, calculated from the expression:

Mc = FabM2

$$F_{ba} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1.0$$

Where:

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4$$

$$P_c = \frac{\pi^2 EI}{(H^{'})^2}$$

$$EI = 0.4\frac{E_c I_g}{1+u}$$

When the action of the dead load is considered and lives or it will be the relation between the alive load maintained and to the axial load of design produced by dead load and alive load. When the died lading is considered, it lives and accidental, or it will be the relation between the axial load of design produced by dead load and alive load maintained and the axial load of design produced by died load, lives and injures

Example 10

Nonsway frame

Figure 10.2 shows and elevation view of a multistory concrete frame, with beams of 120 cm wide and 30 cm height. The clear height of columns is 400 cm. The interior columns are tentative dimensioned 46x46 cm. The frame is effectively braced against sway bys stair and elevator shafts (not shown in the figure). The structure will be subjected to vertical dead and live loads. An analysis of first order indicated the following loads acting on the column C3:

Dead loads	Live loads
P = 104 ton	P = 78 ton
M2 = 0.276 ton-m	M2 = 14.50 ton-m
M1 = -0.276 ton-m	M1 = 13.80 ton-m

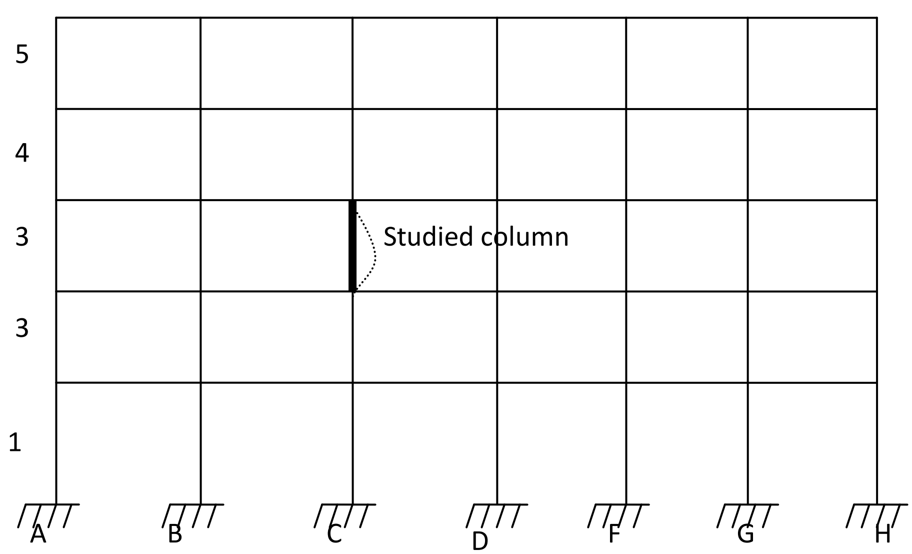


Figure 10.11. Elevation view.

General procedure for the solution:

The First step is to determine whether structure is none sway or sway a frame.

The Second step is to determine whether the effect of slenderness can be neglected or not.

The Third step is to determine if we can use the Magnified Moment Method (MMM) or a Second Order Analysis (SOA).

The Fourth step is to determine the Magnified Moment Method (MMM).

Formulae:

$\frac{kL_u}{r} \leq 34 - 12\frac{M1}{M2}$	$I = \frac{(b)(h)^3}{12}$	$\psi_A = \frac{\left[\Sigma \frac{I_{column}}{L}\right]}{\left[\Sigma \frac{I_{beam}}{L}\right]}$	$r = \sqrt{I/A}$
$\frac{H'}{r} = \frac{kL_u}{r}$	$u = \frac{LL}{LL + DL}$	$E_I = 0.4\frac{(E_c I_y)}{(1 + u)}$	$P_c = \frac{\pi^2 E_I}{H'^2}$
$C_m = 0.6 + 0.4\frac{M1}{M2} \geq 0.4$	$F_{ba} = \frac{C_m}{1 - \frac{p_u}{0.75 p_c}}$	$M_c = F_{ba}(0.276 * M2)$	

Development

Solution:

Step 1

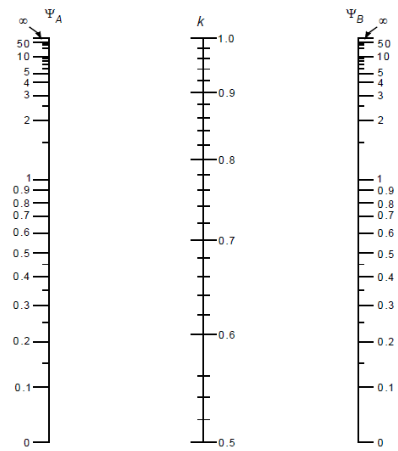
we know that the structure is brace effectively against sway movements by the stairs and the elevator shafts. So the frame is not sway frame.

Step 2

To know if the effect of the slenderness can be neglected we use de following expression:

$$\frac{kL_u}{r} \leq 34 - 12\frac{M1}{M2}$$

Notice that all the variables in the expression are known, except the parameterkL_u, so we need to determine this value with the following Figure:



NTCC, SECTION 1.4.2.2 page. 101

Let use determine $\psi_A and\ \psi_B$

Columns

$$I_{column} = \frac{(46)(46)^3}{12} = 373{,}121.3\ cm2$$

$$L = 400\ cm$$

Beams

$$I_{beam} = \frac{(120)(30)^3}{12} = 270{,}000\ cm2$$

$$L = 732\ cm$$

$$\psi_A = \frac{\left[\Sigma \frac{I_{column}}{L}\right]}{\left[\Sigma \frac{I_{beam}}{L}\right]}$$

$$\psi_A = \psi_B$$

$$\psi_A = \frac{\left[\frac{373{,}121.3}{400}\right](2)}{\left[\frac{270{,}000}{732}\right](2)} = 2.52$$

From figure NTC-2004

K= 0.875

Now let use determine the radius of gyration

$$r = \sqrt{{}^{I}/_{A}}$$

$$r = \sqrt{\frac{\frac{(46)(46)^3}{12}}{(46)(46)}} = 13.27\,m$$

So,

$$\frac{kL_u}{r} \leq 34 - 12\,\frac{M1}{M2}$$

$$\frac{kL_u}{r} = \frac{(0.875)(400)}{13.27} = 26.37$$

$$34 - 12\,\frac{(0.276 + 13.8)}{(0.276 + 14.5)} = 22.56$$

$$26.37 \leq 22.56$$

$$so\ \ we\ need\ to\ consider\ the\ effect\ of\ the\ slenderness$$

Step 3

To determine if we can use the Magnified Moment Method (MMM) or a Second Order Analysis (SOA)

$$\frac{H'}{r} = \frac{kL_u}{r}$$

$$\frac{H'}{r} = 26.22 < 100$$

Now we know that we can use the MMM

Step 4

To determine the Magnified Moment Method (MMM)

$$u = \frac{LL}{LL + DL}$$

$$u = \frac{78}{78 + 104} = 0.428$$

$$E_c = 14{,}000\sqrt{f'_c}$$

$$E_c = 14{,}000\sqrt{250} = 221{,}360\ kg/cm2$$

$$I_{column} = I_y = 373{,}121.3\ cm2$$

$$E_I = 0.4\frac{(E_c I_y)}{(1 + u)}$$

$$E_I = 0.4\frac{(221{,}3600 * 373121.3)}{(1 + 0.428)}$$

$$E_I = 2.3x10^{10}$$

$$P_c = \frac{\pi^2 E_I}{H'^2}$$

$$P_c = \frac{(\pi^2 * 2.3x10^{10})}{(0.87 * 400)^2} = 1{,}853{,}068\ kg$$

$$C_m = 0.6 + 0.4\frac{M1}{M2} \geq 0.4$$

$$C_m = 0.6 + 0.4\frac{(-0.276 + 13.8)}{(0.276 + 14.5)} = 0.96$$

$$C_m = 0.96 \geq 0.4$$

$$F_{ba} = \frac{C_m}{1 - \frac{p_u}{0.75 p_c}}$$

$$F_{ba} = \frac{(0.96)}{1 - \frac{(104{,}000 + 78{,}000)}{0.75(1{,}853{,}068)}} = 1.1$$

$$F_{ba} = 1.1 \geq 1$$

$$M_c = F_{ba}(0.276 * M2)$$

This is the magnified moment.

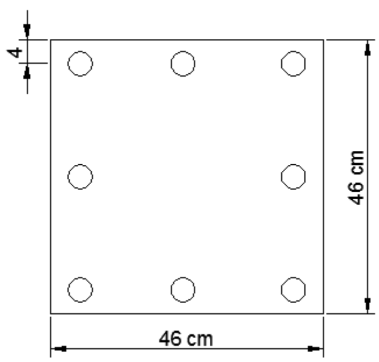
$$M_c = 1.1(0.276 * 14.5) = 16.25\ ton - m$$

Design the Slender column

$$p = 104 + 78 = 182\ ton$$

$$M = M_c = 16.25\ ton - m$$

Let us compute the area for the steel.



$$\frac{d}{h} = \frac{30}{35} = 0.85$$

$$k = \frac{182{,}000}{(0.9)(46)(46)(250)} = 0.38$$

$$R = \frac{1{,}625{,}000}{(0.9)(46)(46^2)(250)} = 0.074$$

With, k and R we can find from the chart “q”. So, with k=0.38 and R=0.074 we found, q=0.10

$$\rho = 0.10\ \frac{170}{4200} = .00404$$

$$A_s = (.00404)(46)(46) = 8.56\ cm2$$

See Problem IX from the annex, page 230

Analysis and design for torsion

Likewise shear analysis much of the knowledge of torsion is empirical.

Torsional effects in reinforced concrete

Cantilevered slab

Edge beam

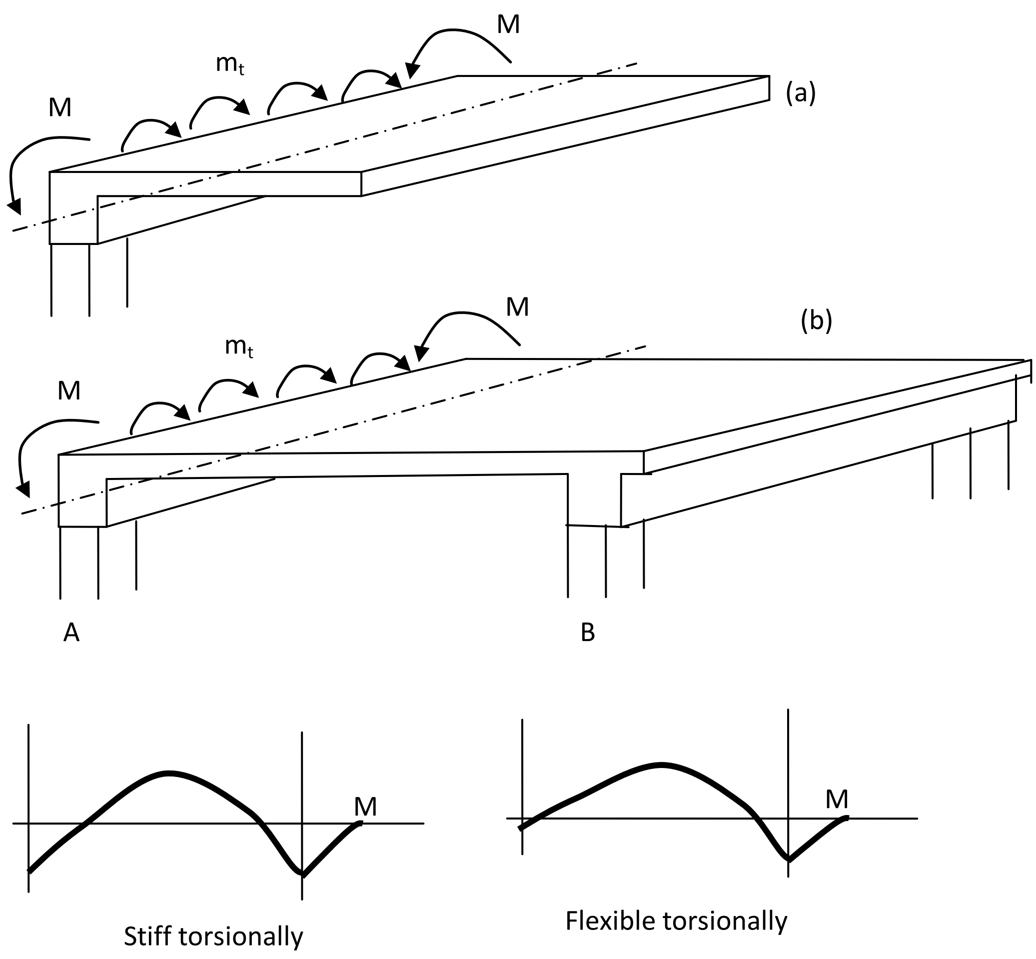


Figure 11.1 Torsional effects on Reinforced concrete.

It is useful in considering torsion to distinguish between primary and secondary torsion in reinforced concrete:

Primary torsion

Exists when the external load has no alternative load path but must be supported by torsion

- Torsional moments can be found form the equilibrium of the structure.
- Can caused the failure of the structure
  Cantilevered slab. Figure 11.1a.

Secondary torsion

- Arises from the requirements of continuity.
- For this case the torsional moments cannot be found based on static equilibrium alone.
- Will not cause the failure of the structure.

Figure 11.1b.

Torsion in plane concrete members

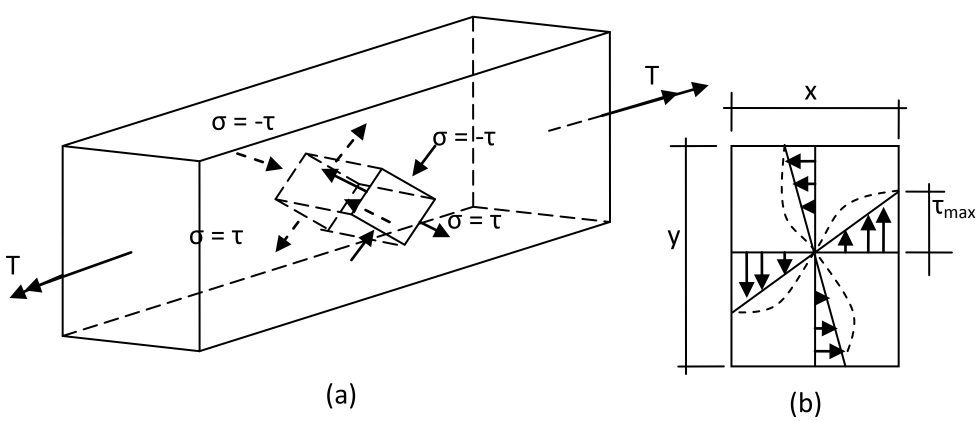


Figure 11.2 Stresses caused by torsion

Figure 2 shows a portion of a prismatic member subjected to equal and opposite torques T at the ends.

If the material is elastic the shear stresses are distributed like in Figure 2b, solid line.

If the material deforms inelastically, as expected from concrete, the stress distribution is closer to that shown by the dashed line.

Failure mode for non reinforced members

Failure is initiated when one crack forms in one the largest side of the beam. Then it expands, rapidly, to the smallest sides and, finally, the failure occurs by crushing in the other largest side of the beam.

The failure is similar to that of a beam in flexural loads.

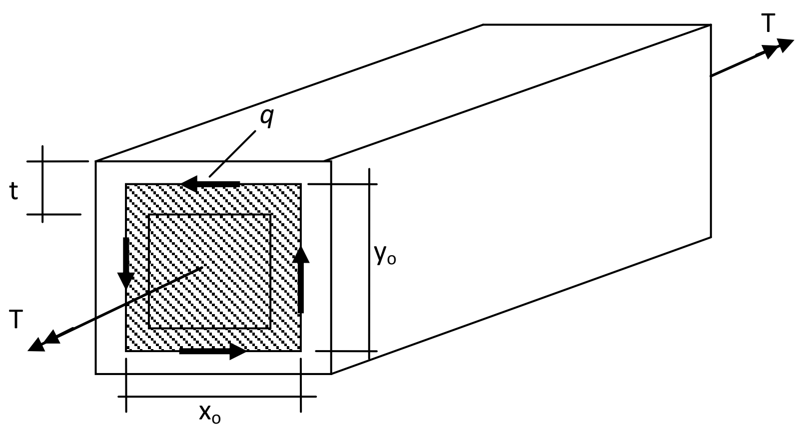


Figure 11.3 Thin-walled tube under torsion

Using this analogy, the shear stresses are treated as constants over a finite thickness t around the perimeter of the tube. As shown in figure 3.

Within the walls of the tube, torque is resisted by the shear flow q, which has units of force per unit length.

In this analogy q, is treated as a constant around the perimeter of the tube.

The resultants of the individual components of shear flow are located within the walls of the tube and act along lengths yo, in verticals walls and along xo in horizontal walls.

The relationship between the applied torque and the shear flow can be obtained by summing the moments about the axial centerline of the tube, giving

$$T = 2qx_o \frac{y_o}{2} + 2qy_o \frac{x_o}{2}$$

$$T = 2qx_o y_o$$

The product X_0 Y_0 represents the area enclosed by the shear flow path A_0, givin

$$T = 2qA_o$$

And

$$q = \frac{T}{2A_o}$$

Note that, although A0 includes the area of the hollow box as well as solid sections, so it applies for solid sections too.

For a tube wall thickness t, the unit shear stress acting within the walls of the tube is:

$$\tau = \frac{q}{t} = \frac{T}{2A_o t}$$

From Figure 2 we know that:

$$\tau = \sigma$$

Substituting τ = f't (strength tension of concrete type 1)

$$f'_t = 1.3\sqrt{f*_c} = \frac{q}{t} = \frac{T}{2A_o t}$$

$$1.3\sqrt{f*_c} = \frac{T}{2A_o t}$$

From last expression T is:

$$T_{cr} = 1.3\sqrt{f*_c}(2A_o t)$$

The value of t can be approximated as a fraction of the ratio Acp/Pcp where Acp is the full concrete cross section, and Pcp is the perimeter of the cross section.

For solid members t is approximately ¼ of the minimum width and using a member with a width-to-depth ratio of 0.5 yields a value:

$$A_o = \frac{2}{3}A_{cp}$$

And

$$t = \frac{3}{4}\frac{A_{cp}}{P_{cp}}$$

Substituting these values in Tcr equation:

$$T_{cr} = 1.3\sqrt{f*_c}\,\frac{A_{cp}^{\;2}}{P_{cp}}$$

Applying a security factor Tcr can be calculated:

$$T_{cr} = F_R\sqrt{f*_c}\,\frac{A_{cp}^{\;2}}{P_{cp}}$$

Example 11:

Calculate the strength to torsion of the following concrete section:

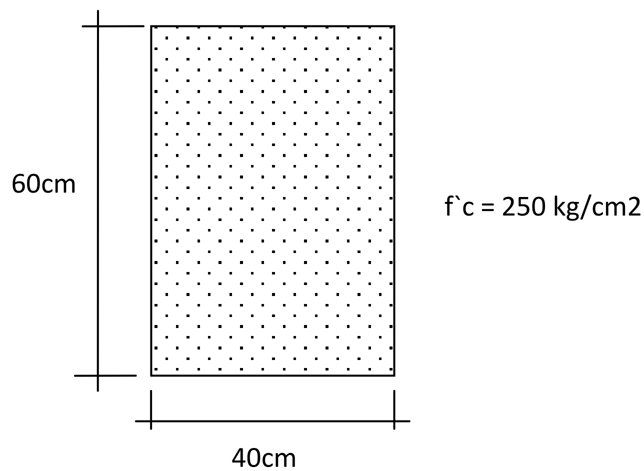


Figure 11.4

General procedure

The First step is to calculate the area and the perimeter from the concrete section.

The Second step is to calculate the torsion.

Formulae:

$$T_{cr} = F_R\sqrt{f^*{}_C}\left(\frac{A_{cp}{}^2}{P_{cp}}\right) \qquad F^*{}_C = 0.8\,f'{}_c$$

STEP 1

$$A_{cp} = 40 * 60 = 2400\ cm2$$

$$P_{cp} = 200\ cm$$

STEP 2

$$T_{cr} = F_R \sqrt{f^*{}_C} \left(\frac{{A_{cp}}^2}{P_{cp}}\right)$$

$$F^*{}_C = 0.8\, f'_c$$

$$T_{cr} = (0.8)\sqrt{200}\left(\frac{(2400)^2}{200}\right) = 325{,}834.8\ kg/cm$$

$$T_{cr} = 325.8\ ton/c$$

See Problem X from the annex, page 231

Torsion in reinforced concrete members

To resist torsion for values of T above Tcr, reinforcement must be added, this consist of closely spaced stirrups and longiditunal bars.

The cracks form a spiral pattern, as shown in Figure 11.5.

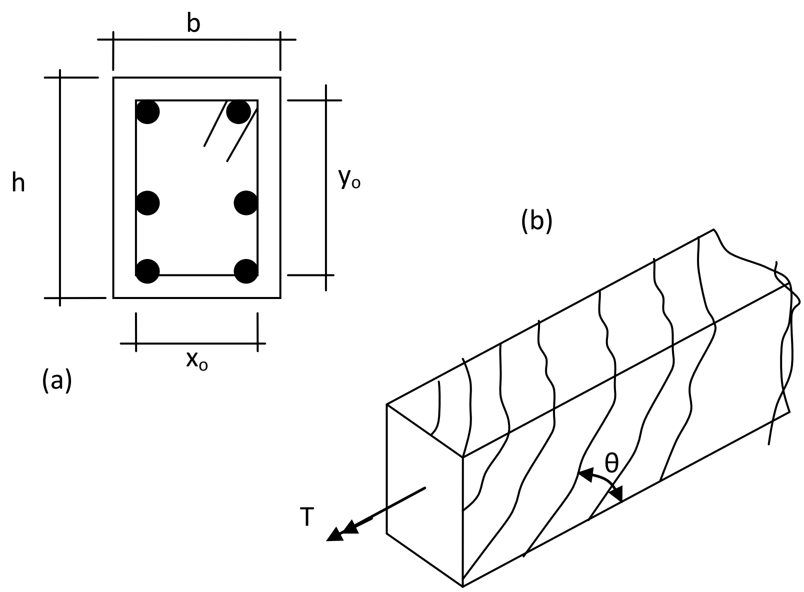


Figure 11.5 Reinforced concrete beam in torsion. (a) Reinforcement for torsion (b) Torsion cracks

Reinforced members

Upon cracking, the torsional resistance of the concrete drops to about half of that of the uncracked member, immediately, the torsions effects are resisted by the reinforcement. This can be seen in Figure 11.6.

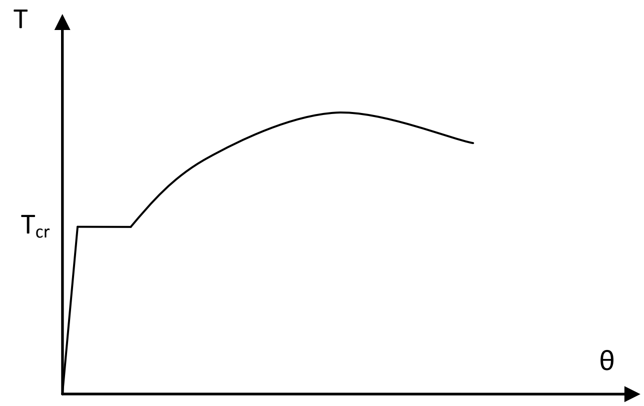


Figure 11.6. Torque-twist curve in reinforced concrete.

Tests show that, after cracking, the area enclosed by the shear path is defined by the dimensions xo and yo measured to centerline of the outermost closed transverse reinforcement (Figure 4a). These dimensions define the gross area Aoh = xo yo and the shear perimeter Ph = 2(xo + yo).

Space truss analogy (Figure 11.7)

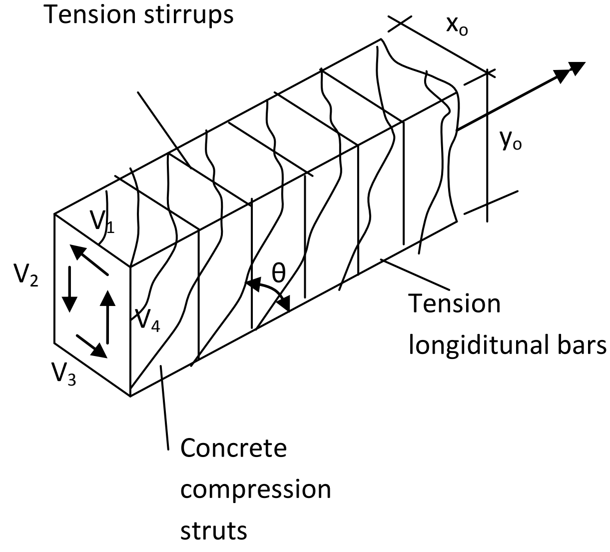


Figure 11.7. Space truss analogy

Torsion transverse reinforcement

With reference to Figure 11.8a, the torsional resistance provided by a member with a rectangular cross section can be represented as the sum of the contributions of the shears in each of the four walls of the equivalent hollow tube.

The contribution of the shear acting in the right-hand vertical wall is:

$$T_4 = \frac{V_4 x_o}{2} \quad \text{(a)}$$

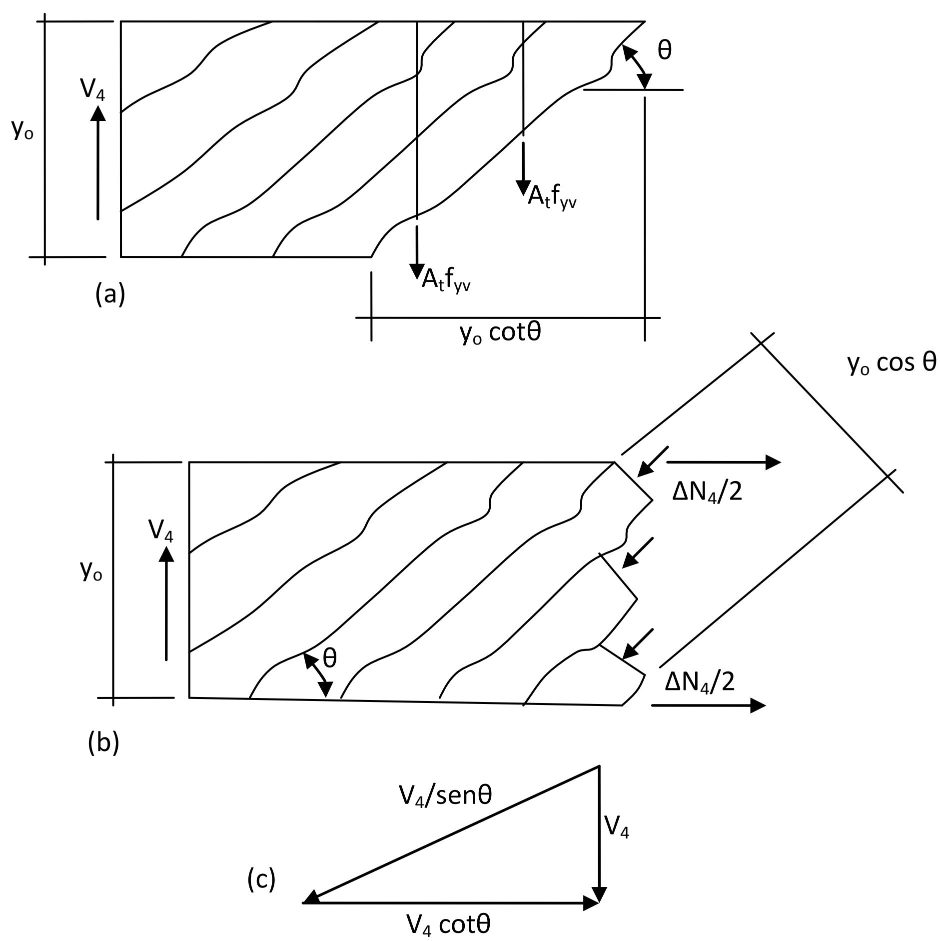


Figure 11.8 Basis for torsional design: (a) Vertical tension in stirrups (b) diagonal compression in vertical wall of beam (c) equilibrium diagram of forces due to shear in vertical wall

From Figure 11.8a and assuming that the vertical steel is yielding:

$$V_4 = A_t f_{yv} n$$

Where:

At = area of one leg of a closed stirrup

fyv = yield strength of transverse reinforcement

n = number of stirrups intercepted by one torsional crack

Considering that the horizontal projection of the crack Y_0cotθ and s is the spacing of the:

$$n = \frac{y_o \cot\theta}{s}$$

$$V_4 = \frac{A_t f_{yv} y_o}{s} \cot\theta$$

So the contribution of the shear force V4 to resist torsion is:

$$T_4 = \frac{A_t f_{yv} y_o x_o}{2s} \cot\theta$$

The sum of the four walls gives the total nominal capacity which is:

$$T_n = \sum_{i=1}^{4} T_i = \frac{A_t f_{yv} y_o x_o}{2s} \cot\theta$$

$$T_n = \frac{2A_t f_{yv} y_o x_o}{s} \cot\theta$$

As

$$x_o y_o = A_{oh};$$

$$T_n = \frac{2A_t f_{yv} A_{oh}}{s} \cot\theta$$

$$T_R = \frac{F_R 2A_t f_{yv} A_{oh}}{s} \cot\theta$$

Calculating At from last expression:

$$A_t = \frac{T_R s}{F_R 2A_{oh} f_{yv} \cot\theta}$$

It is known by experimentation that the real value of Aoh must be reduced in 15 %. Thus: Ao = 0.85Aoh.

$$A_t = \frac{T_R s}{F_R 2A_o f_{yv} \cot\theta}$$

Where At is the additional transverse reinforcemet to resist torsion. This reinforcement can be combined with others.

<u>Torsion longitudinal reinforcement</u>

From Figure 11.8(b). The horizontal component of the compression force must be equilibrated by the tension in the steel ΔN4. From Figure 11.8c we can find the value of ΔN4.

$$\Delta N_4 = V_4 \cot\theta = \frac{A_t f_{yv} y_o}{s} \cot^2\theta$$

Again, summing the four sides it yields:

$$\Delta N_i = \sum_{i=1}^{4} \Delta N_i = \frac{A_t f_{yv}}{s} 2(x_o + y_o)\cot^2\theta$$

Note that 2(xo + yo) is the perimeter of the area enclosed by the transverse reinforcement Ph and substituting:

$$\Delta N = \frac{A_t f_{yv} P_h}{s} \cot^2\theta$$

A quantity of longitudinal steel must be provided to resist the force ΔN. Considering that the steel is yielding:

$$A_{st} f_y = \frac{A_t f_{yv} P_h}{s} \cot^2\theta$$

And the additional longitudinal steel is:

$$A_{st} = \frac{A_t}{s} P_h \frac{f_{yv}}{f_y} \cot^2\theta$$

Where:

Ast = Additional longitudinal steel

Ph = Perimeter of the area enclosed by the transverse reinforcement

fyv = Yield strength of transverse steel (no larger 4200kg/cm2)

fy = Yield strength of longitudinal steel

θ = Angle 45°

s = space between stirrups

This steel must be placed at the perimeter at a maximum spacing of 300 mm.

Chapter III

Serviceability

III. SERVICEABILITY

SYNOPSIS

When we are designing the serviceability, we must be aware that the initial structure can perform its intended mission; we have to remember that the intended function must be completed under the day service loads. We have some aspects that we have to check in the designing of serviceability. The deflection must not be excessive, Cracks must be controlled and the structure shouldn't suffer excessive vibration. Another aspect is the shrinkage which causes time-dependent cracking and the consequence of these cracks is the reduction of the stiffness in the structure, and this is a detrimental factor in the design of the serviceability.

Going back to the deflection problems, we found that there are three main types that may affect the serviceability structure:

1. Excessive deflections causes aesthetic problems (visual hogging and ponding of water on roofs)

2. Excessive deflections results in damage to the structural element attached to the member (cracking of masonry walls or other partitions, damage in ceiling or floor, not functional windows and doors)

3. Due to the insufficient stiffness cause unpleasant moments to the occupants (springy vertical motion of floor systems)

SERVICEABILITY

Due to the low tensile resistance of concrete, the reinforced concrete members crack.

All Reinforced concrete beams crack, generally starting at loads well below service level, and possibly even prior to loading due to restrained shrinkage. Flexural cracking due to loads is not only inevitable, but actually necessary for reinforcement to be used effectively

Cracking of concrete is a ramdom process, highly variable and influenced by many factors.

Because of the complexity of the problem, present methods for predicting crack widths are based primarily in test observations.

Most equations that have been developed predict the probable maximum crack width

Two are the reasons to control the width of cracks: appearance and steel corrosion (GC).

- Cracking mechanism
- Classic mechanism
- Basic assumptions
- Tension stress distribution in an effective area around the reinforcement bars
- Bond forces are developed around reinforcement bars

1.- At the moment of applying tension forces the first cracks appear (1) at the most weak zones of concrete (sections A)

At the sections the steel stresses are fs= T/As. At the intermediate sections this stress is lowered by the transmission of this force to the concrete by bond forces

2.- If the tension force being transmitted to the concrete is larger than the one it can resist, then new cracks appear (2, sections B).

This process continues until the force being transmitted to the concrete is lower than that it can resist (Figure 12.1).

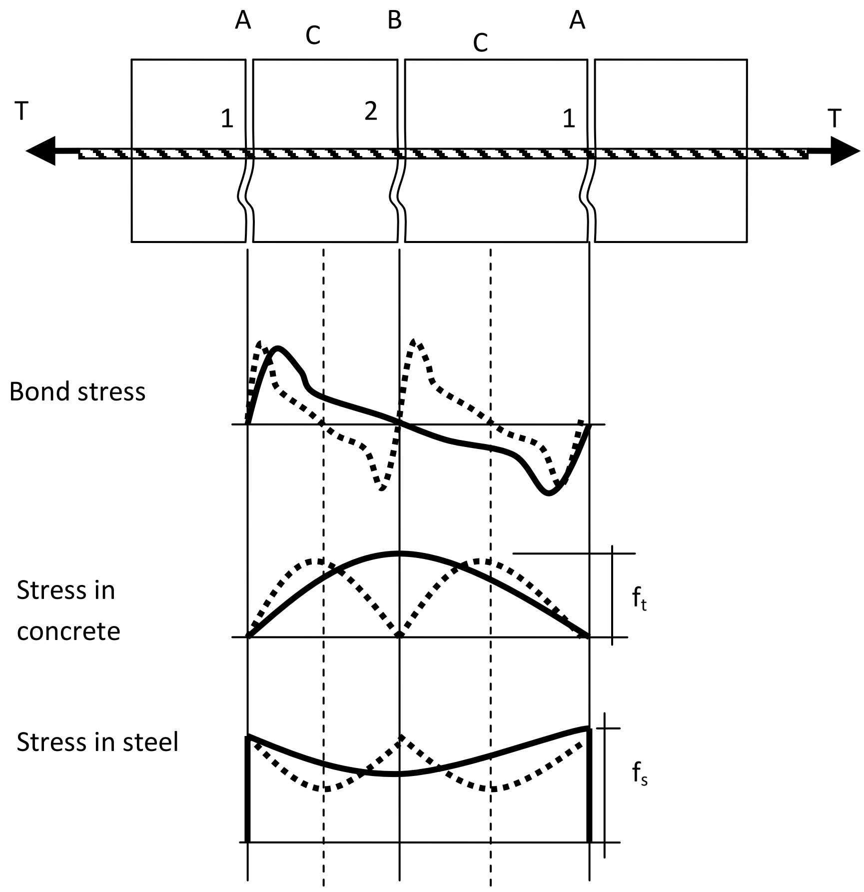


Figure 12.1 Mechanism of cracking

<u>Variables affecting width of cracks</u>

In general, beams with smooth round bars will display a relatively small number of rather wide cracks in service, while beams with good slip resistance ensured by proper surface deformations on the bars will show a larger number of very fine, almost invisible cracks.

Increasing the cover increases the spacing of cracks an also increased the width.

Increasing the stress in steel will increase the width of cracks.

Generally to control cracking, it is better to use larger number of smaller-diameter bars than to use the minimum number of larger bars (Nilson)

Expressions from the NTC-2004 to predict the probable crack width

$$w_{\max} = f_s \sqrt[3]{d_c A}\,\frac{h_2}{h_1}\,x10^{-6} \quad \text{(cm)}$$

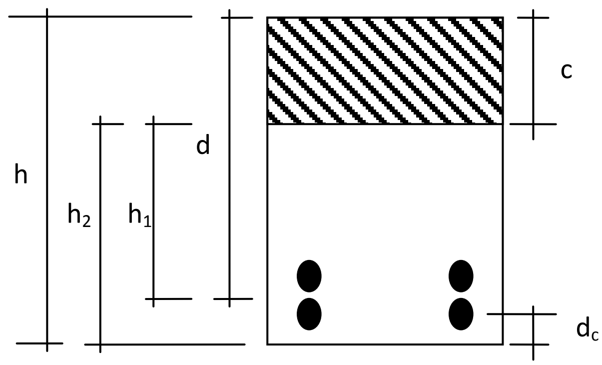
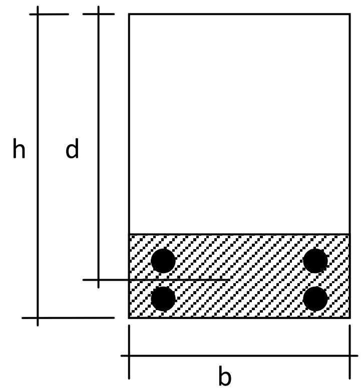


Figure 12.2. Variables to predict the probable crack width

A= tension concrete area around the reinforcement bars (A = Ae/N; N = Number of bars).

When the reinforcement consist of a several number of different diameter bars, a number of equivalent bars should be calculated dividing the total steel area between the area of the larger diameter bar

Determination of effective concrete area around steel bars Ae (Figura 12.3)



$A_e = 2b(h-d)$

Figure 12.3 Effective area

The steel stress can be calculated from the following expression:

$$f_s = \frac{M}{A_s z}$$

Where z may be

$$\frac{7}{8}d \; ;$$

or can be calculated from the cracked section. It is suggested to use the latter value.

Cracked section

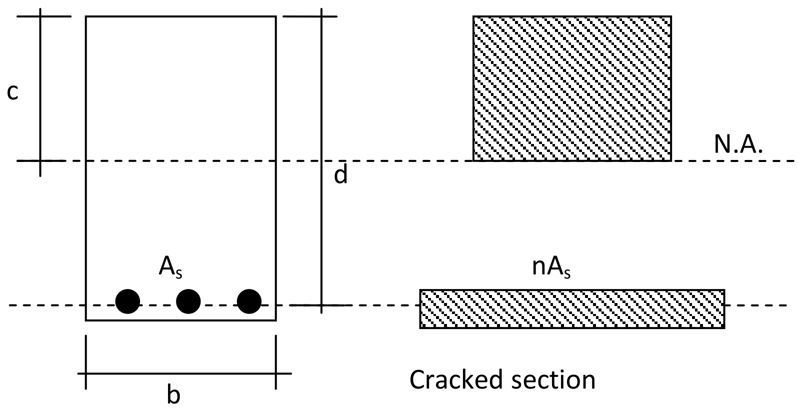


Figura 12.4 Artificio de la sección transformada

$$n = \frac{E_s}{E_c}$$ (Modulus relationship)

Es = Modulus of elasticity of steel = 2x106 (kg cm-2)

Ec = Modulus of elasticity of concrete = $14000\sqrt{f'_c}$ (kg cm-2).

From Figure 12.1 we calculated the value of c

From the primary moment

$$bc\left(\frac{c}{2}\right) = nA_s(d-c)$$

$$\frac{bc^2}{2} + nA_s c - nA_s d = 0$$

Example 12

Calculate the probable crack width.

a) Modify the number of bars and see the effect on the width of the crack

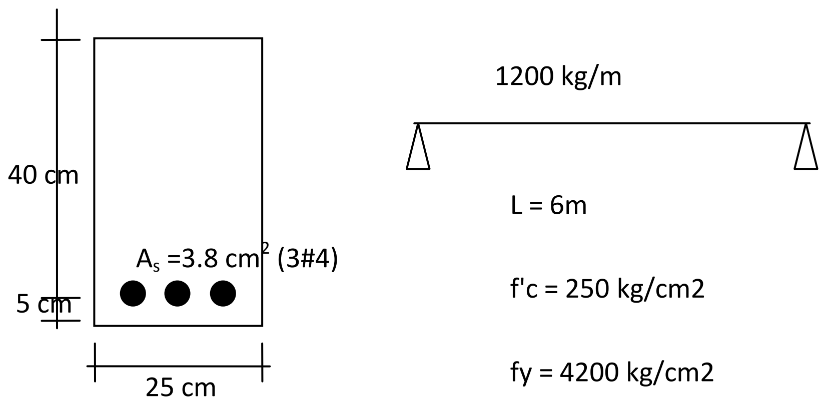


Figure 12.5

General Procedure of the solution:

The First step is to compute the crack section for computing "c"

The Second step is calculate the Tension concrete area around the reinforcement bars

The Third step is calculate the Steel stress

The Fourth step is calculate the crack.

Formulae

$$\frac{bc^2}{2} + nA_sc - nA_Sd = 0 \qquad A = \frac{A_e}{N} \qquad f_s = \frac{M}{A_s z} \qquad M = \frac{wl^2}{8}$$

$$z = h1 + \frac{c}{2} \qquad w = f_s \sqrt[3]{d_c A}\frac{h2}{h1} \qquad w_{max} = f_s \sqrt[3]{d_c A}\frac{h2}{h1} x10^{-6}$$

Solution:

STEP 1

$$n = \frac{2x10^6}{14{,}000\sqrt{250}} = 9.03$$

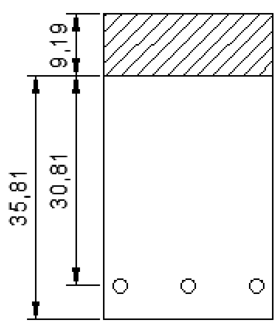
$$\frac{bc^2}{2} + nA_s c - nA_S d = 0$$

$$\frac{25c^2}{2} + (9.03 * 3.8)c - (9.03 * 3.8 * 40) = 0$$

$$12.5c^2 + 34.314c - 1372.56 = 0$$

$$c = 9.19\ cm$$

STEP 2



$$h1 = 30.81\ cm$$

$$h2 = 35.81\ cm$$

$$A_e = (2 * 25 * 5) = 250\ cm2$$

Figure 13.10

Tension concrete area around the reinforcement bars

$$A = \frac{A_e}{N}$$

$$A = \frac{250}{3} = 83.33\ cm2$$

STEP 3

Steel stress

$$f_s = \frac{M}{A_s z}$$

$$M = \frac{wl^2}{8}$$

$$M = \frac{(1200)(6)^2}{8} = 5400\ kg - m$$

$$M = 540{,}000\ kg - cm$$

$$z = h1 + \frac{c}{2}$$

$$z = 30.81 + \frac{9.19}{2}$$

$$z = 35.4\ cm$$

So, f_s

$$f_s = \frac{540{,}000}{(3.8 * 35.4)} = 4{,}014.27\,\frac{kg}{cm2}$$

STEP 4

Expression from NTC 2004

$$w = f_s \sqrt[3]{d_c A}\,\frac{h2}{h1}$$

$$w = 4{,}014.27 * \sqrt[3]{(5 * 83)}\,\frac{35.81}{30.81}$$

$$w = 34{,}000$$

This condition pass the categoriesA,B TABLE 6.77 NTCC-2004

The probable crack width is compute as follow (note that this new expression is multiply by the factor 10^{-6} so the expression gives the probable crack width in cm

$$w_{max} = f_s \sqrt[3]{d_c A} \frac{h2}{h1} x10^{-6}$$

$$w_{max} = 4{,}014.27 * \sqrt[3]{(5*83)} \frac{35.81}{30.81} x10^{-6}$$

$$w_{max} = 0.034cm = 0.34\ mm$$

See Problem XI from the annex, page 232

Serviceability

On a structure deflections and cracking must be limited to values that do not affect the use of the structure (NTC-2004).

Criteria must be established on the acceptable values for deflections.

The restraint of deflections is important, think of two points:

First: Large deflections of a member may cause damage on other structure parts or on not structural members, which is more often.

Second: the human response to large deflections

Deflections

Total deflection must be the sum of immediate and long term deflections

Immediate deflections

These deflections take place at the moment of loading. They must be calculated from the ordinary methods to calculate elastic deflections, and the effective moment of inertia Ie calculated from the following equation:

$$I_e = \left(\frac{M_{ag}}{M_{max}}\right)^3 I_g + \left[1 - \left(\frac{M_{ag}}{M_{max}}\right)^3\right] I_{ag}$$

Where:

$$M_{ag} = \frac{\overline{f_f} I_g}{h_2}$$ (Cracking moment)

From the equation (ρ = Mc/I)

Mmax = Maximum flexural moment.

h2 = Distance from the neural axis to the most stressed fiber (gross section)

Ig = Moment of inertia of the non cracked section

Iag = Moment of inertia of the cracked section

$\overline{f_f}$ = Rupture modulus of concrete = $2\sqrt{f'_c}$ type 1 concrete

As an option and as a simplification one can use the moment of inertia of the cracked section Iag instead of the effective moment of inertia Ie.

In continuous members the moment of inertia that must be calculated will be the average computed from the following expression:

$$I = \frac{I_1 + I_2 + 2I_3}{4}$$

Where I1 and I2 are the extremity moments of inertia and I3 of the section at the center

If the element is continuous only in one side the moment of inertia corresponding to the side will be taken equal to zero and the denominator will be three.

Cracked section

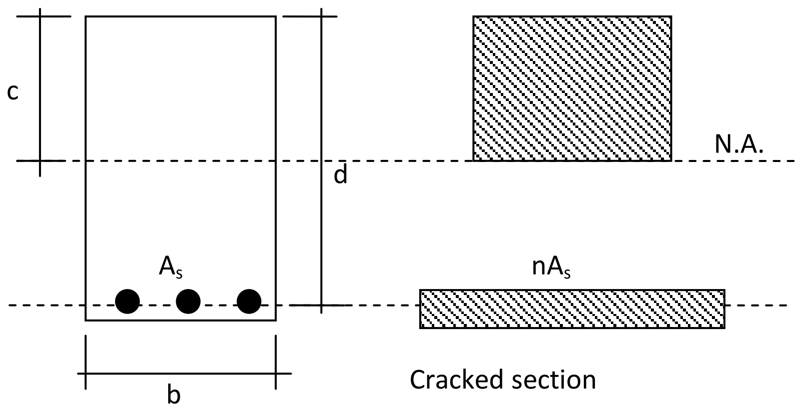


Figura 12.6 Artificio de la sección transformada

$$n = \frac{E_s}{E_c} \text{ (Modulus relationship)}$$

Es = Modulus of elasticity of steel = 2x106 (kg cm-2)

Ec = Modulus of elasticity of concrete = $14000\sqrt{f'_c}$ (kg cm-2).

From Figure 11.1 we calculated the value of c

From the primary moment

$$bc\left(\frac{c}{2}\right) = nA_s(c-d)$$

$$\frac{bc^2}{2} + nA_s c - nA_s d = 0$$

Compute “c”

Once computed c one can compute the moment of inertia of the cracked section

Long term deflections

The long term deflection will be calculated multiplying the immediate deflection by the factor:

$$\frac{2}{1+50p'} \text{ (Concrete type 1)}$$

$$\frac{4}{1+50p'} \text{ (Concrete type 2)}$$

Where p’ is the reinforcement ratio at compression.

The same criteria for the moment of inertia is applied to the calculus of p’

Beam with simple supports

$$\delta = \frac{5wL^4}{384EI}$$

Beam with continuous supports

$$\delta = \frac{wL^4}{384EI}$$

Restriction

L/240 +.5 (cm)

Example 13

Compute the immediate and long term deflections, with Iag and Ie.

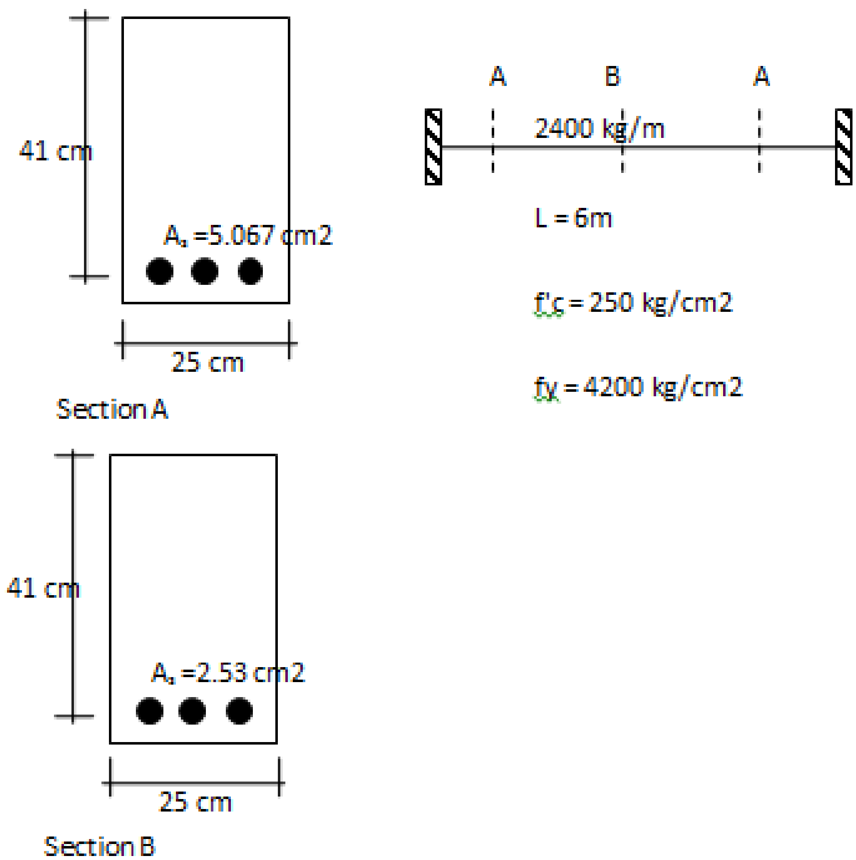


Figure 12.7

General procedure for the solution:

The First Step is to compute the effective moment of inertia of each section

The Second step is to compute the average moment of inertia

The Third step is to compute the Immediate deflection (δ) and long term deflection

The Fourth step is to check the total deflections with the admissible deflection

Formulae:

$M_{ay} = \frac{\bar{f}_f I_y}{h_2}$	$\bar{f}_f = 2\sqrt{f'_c}$	$M_{max} = \frac{wl^2}{12}$	$\frac{bc^2}{2} + nA_s c - nA_S d = 0$
$I_e = \left(\frac{M_{ay}}{M_{max}}\right)^3 * I_y + \left[1 - \left(\frac{M_{ay}}{M_{max}}\right)^3\right] * I_{ag}$		$\delta = \frac{wl^4}{384EI}$	$I = \frac{(I_{e1} * 2) + (I_{e2} * 2)}{4};$
$long\ term\ \delta = factor * inmediate\ \delta$		$p' = \frac{A'_S}{(b)(d)};$	$fact = \frac{2}{1 + 50(p')}$
$\delta_{total} = inmediate\delta + long\ term\delta$		$\delta = \frac{l}{240} + 0.5$	

Solution:

STEP 1

Section A

Cracked moment

$$M_{ay} = \frac{\bar{f}_f I_y}{h_2}$$

$$\bar{f}_f = 2\sqrt{f'_c}$$

$$\bar{f}_f = 2\sqrt{250} = 31.62\ kg/cm2$$

$$I_y = \frac{(25)(45)^3}{12} = 189{,}843.75\ cm4$$

$$h_2 = \frac{45}{2} = 22.5\ cm$$

$$M_{ay} = \frac{(31.61 * 189{,}843.75)}{22.5} = 266{,}817.17\ kg - cm$$

Ultimate Moment

$$M_{max} = \frac{wl^2}{12}$$

$$M_{max} = \frac{(2400)(60)^2}{12} = 720{,}000\ kg - cm$$

Cracked section

$$n = \frac{2x10^6}{14{,}000\sqrt{250}} = 9.03$$

$$\frac{bc^2}{2} + nA_s c - nA_S d = 0$$

$$\frac{25c^2}{2} + (9.03 * 5.067)c - (9.03 * 5.067 * 41) = 0$$

$$12.5c^2 + 45.76c - 1875.96 = 0$$

$$c = 10.55\ cm$$

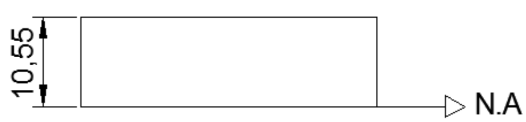


Figure 12.8

$$41 - 10.55 = 30.45\ cm$$

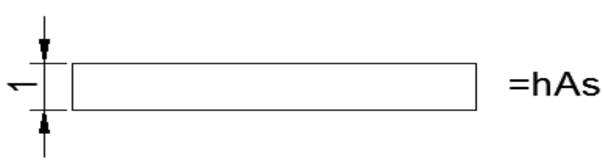


Figure 12.9

$$hA_s = (9.03)(5.067) = 45.75\ cm2$$

Let us compute I_{ag}

$$I_{ag} = \frac{25(10.55)^3}{3} + 45.75(30.45)^2 + \frac{1(45.75)^3}{12}$$

$$I_{ga} = 60{,}184.66\ cm4$$

$$I_e = \left(\frac{M_{ay}}{M_{max}}\right)^3 * I_y + \left[1 - \left(\frac{M_{ay}}{M_{max}}\right)^3\right] * I_{ag}$$

$$I_e = \left(\frac{266{,}817.17}{720{,}000}\right)^3 * 189{,}843.75 + \left[1 - \left(\frac{266{,}817.17}{720{,}000}\right)^3\right] 60{,}184.66;$$

$$I_e = 66{,}783.18\ cm4$$

Section B

$$\frac{25c^2}{2} + (9.03 * 2.53)c - (9.03 * 2.53 * 41) = 0$$

$$12.5c^2 + 22.8459c - 936.6819 = 0$$

$$c = 7.79\ cm$$

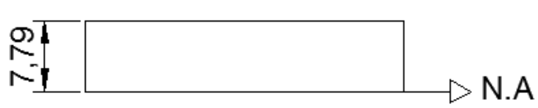


Figure 12.10

$$41 - 7.79 = 33.21\ cm$$

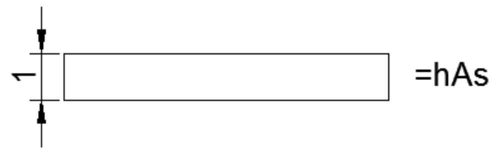


Figure 12.11

$$hA_s = (9.03)(2.53) = 22.84\ cm2$$

Let us compute I_{ga}

$$I_{ga} = \frac{25(7.79)^3}{3} + 22.84(33.2)^2 + \frac{1(22.84)^3}{12}$$

$$I_{ga} = 30{,}122.64\ cm4$$

$$I_e = \left(\frac{266{,}817.17}{720{,}000}\right)^3 * 189{,}843.75 + \left[1 - \left(\frac{266{,}817.17}{720{,}000}\right)^3\right] 30{,}122.64;$$

$$I_e = 38{,}248.9 cm4$$

STEP 2

The average moment of inertia:

$$I = \frac{(I_{e1} * 2) + (I_{e2} * 2)}{4};$$

$$I = \frac{(66{,}783.18 * 2) + (38{,}248.9 * 2)}{4};$$

$$I = 52{,}516.04\ cm4$$

STEP 3

Immediate deflection (δ)

$$\delta = \frac{wl^4}{384EI}$$

$$\delta = \frac{(24)(600)^4}{384\left(14{,}000\sqrt{250}\right)(52{,}516.04)};$$

$$\delta = 0.69\ cm$$

STEP 4

Let us compute long term deflections

Consider a uniform quantity of compression steel that $A'_S = 2.4\ cm2$ for the complete beam.

$$p' = \frac{A'_S}{(b)(d)};$$

$$p' = \frac{2.4}{(25)(41)} = 0.0023$$

The factor will be

$$fact = \frac{2}{1 + 50(p')}$$

$$fact = \frac{2}{1 + 50(0.0023)} = 1.79$$

Long term deflection

$long\ term\ \delta = factor * inmediate\ \delta$

$long\ term\ \delta = (1.79 * 0.69)$=1.23 cm

Total deflection.

$$\delta_{total} = inmediate\delta + long\ term\delta$$

$$\delta_{total} = 0.69 + 1.23$$

$$\delta_{total} = 1.92\ cm$$

So, let us check the restriction for the deflection (NTC section 3.2.1-pag2 123)

$$\delta_{allowable} = \frac{l}{240} + 0.5$$

L= 600 cm

$$\delta_{allowable} = \frac{600}{240} + 0.5$$

$$\delta_{allowable} = 3.0\ cm$$

$$\delta_{allowable} > \delta\ ok$$

The restriction say, if deflection is smaller than the restriction, so we need to use, the last deflection, in this case is 3.0 cm

See Problem XII from the annex, page 233

Chapter IV

Bond and Anchorage

IV. Bond and Anchorage

SYNOPSIS

The reinforced concrete is a material consisting of concrete and reinforcement, so basically this means that hence concrete and re-bars must behave as an integrated manner that will provide the bond and the anchorage. Beyond the basic requirement applies to the RC material which has a high strength as routine, the bond and the anchorage capacity must increase along with the increase of the steel stress, because of the arrangement of the similar bars as the conventional reinforced concrete is expected in the new RC structures, the members that are used for building, usually have a very flat layer of concrete covering the axial bars, and sometimes this can cause a splitting failure. But in the other hand the beams and the columns must transfer forces in each other members.

The reinforcement in a reinforcement concrete, such as a steel bar has to take the same deformation as the surrounding concrete, this to prevent slip or separation, maintaining this composite action requires a transfer of load between the concrete and the steel, the direct tension is transferred from the concrete to the steel bar, so as to change the tensile. You can get this load by means of bond, a continuous stress field.

The bond stress varies along the length of the bar, the international codes of specifications use zone of tension instead of the bond stress, and the first objective for safety against a bond failure is to provide a sufficient extension of length.

BOND AND ANCHORAGE

In Reinforced concrete elements bond forces must develop between concrete and the reinforcement bars, otherwise the concrete behavior would be different from the one studied so far.

Figure shows the differences a) bond forces exists d) bond forces do not exist

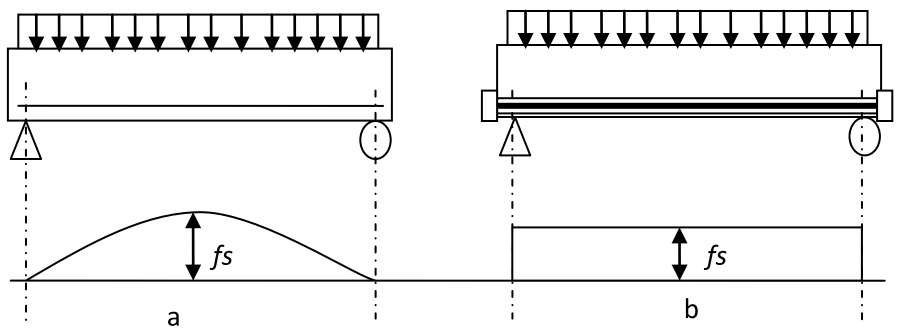


Figure 13.1 Concrete elements with and without bond forces

Physical principles of bond forces

- Chemical adhesion between concrete and steel
- Friction forces developed at the moment of sliding of steel
- Friction at the deformed surface of steel
- Bond and anchorage
- Steel must be embedded so that steel can reached its yielding stress

Consider Figure 13.2

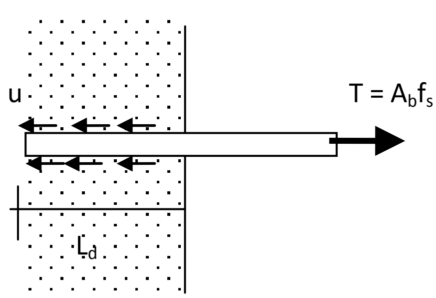


Figure 13.2 Bond forces at the embedment

From equilibrium:

$$u\pi d_b L_d = \frac{\pi d_b^2 f_s}{4}$$

Where:

db = Bar diameter

Ld = Embedded longitude

fs = Stress at steel

u = Average bond force

Finding u from last equation

$$u = \frac{d_b f_s}{4L_d}$$

If u is known and taking fs = fy one can calculate Ld in order for the steel to develop its yielding stress.

$$L_d = \frac{f_y d_b}{4u}$$

Figure 13.3 shows the behavior of bond forces in a pullout test of a smooth bar.

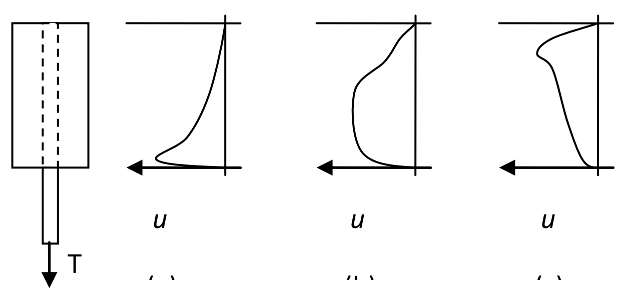


Figure 13.3 Behavior of bond forces in a pullout test of a smooth bar.

Fundamentals of bond strength

Considering a dx longitude (Figure 13.4), the moment in one side is different from the other side and varies in dM.

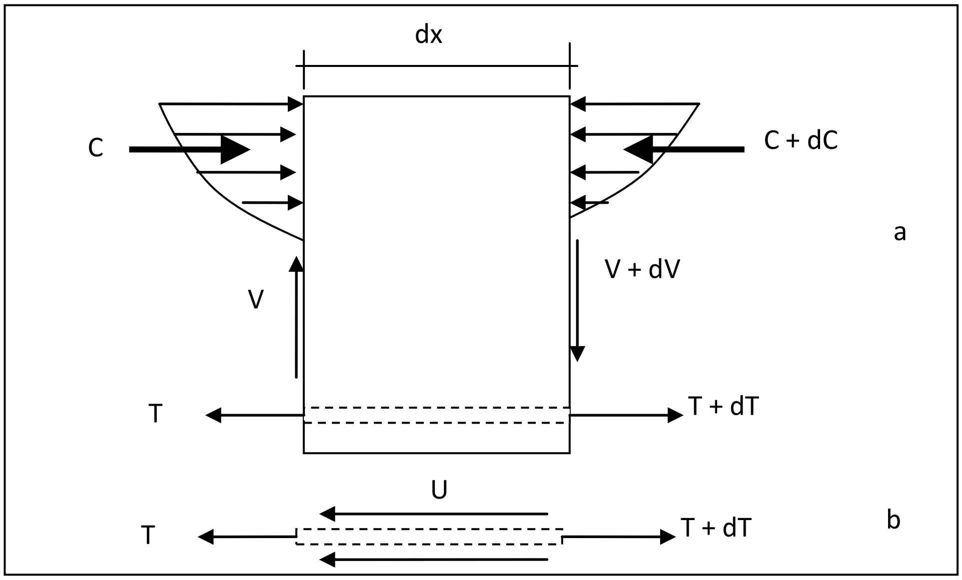


Figure 13.4 Forces acting in a dx beam longitude

Tension in steel can be calculated:

$$T = \frac{M}{z}$$

Thus the tension increment dT in the reinforcement steel caused by the dM is:

$$dT = \frac{dM}{z}$$

This tension increment is resisted by the bond forces acting along dx of the steel bar

From x sum forces (Figure 13.6b):

$$Udx = dT$$

thus

$$U = \frac{dT}{dx}$$

Last expression indicates that local bond force is proportional to the rate of change of bar force along the span.

Substituting dT in last expression:

$$U = \frac{1}{z}\frac{dM}{dx} = \frac{V}{z}$$

Note that the new expression applies to tension bars in a concrete zone that is assume to be completely cracked, with the concrete resisting no tension. It applies, therefore, in simple spans or in continuous spans.

It does not apply to compression reinforcement

Actual distribution of flexural bond force

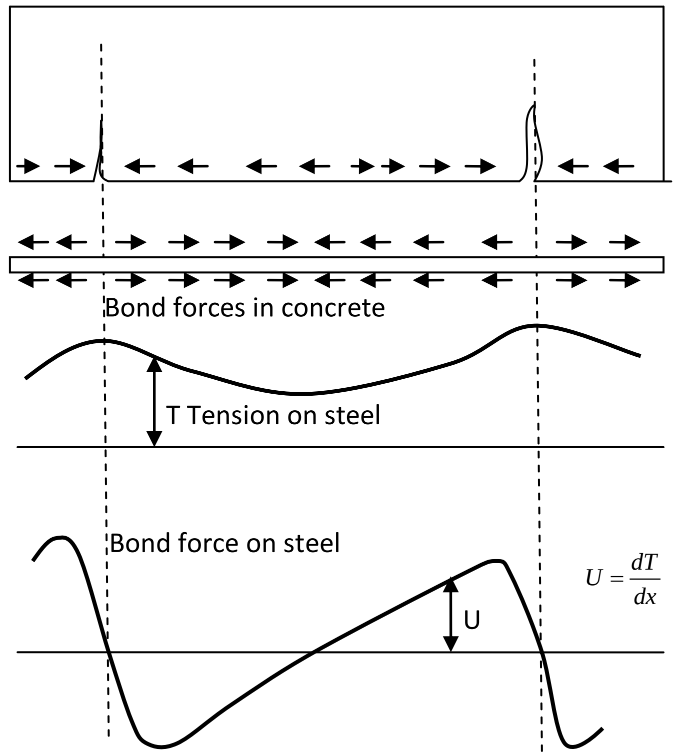


Figure 13.5 Variation of steel and bond forces in a reinforced concrete member subjected to pure bending

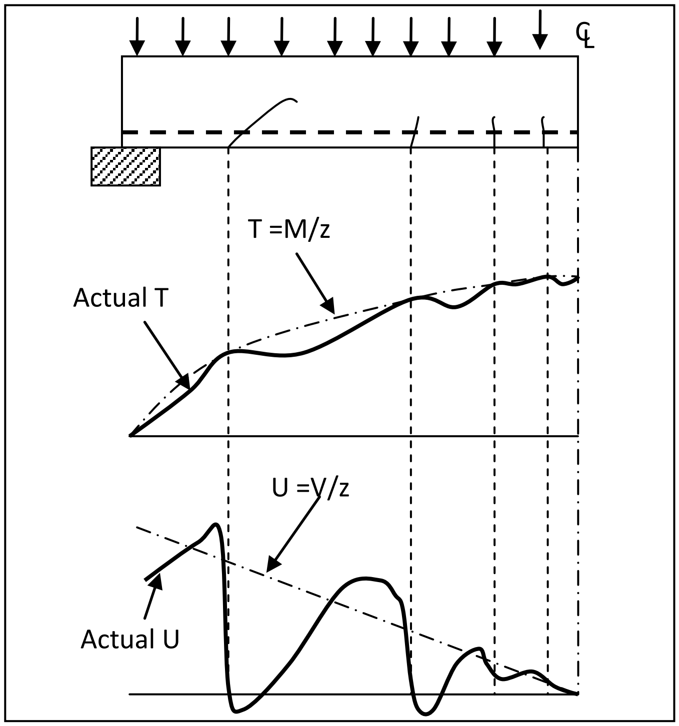


Figure 13.6 Effect of flexural cracks on bond forces in a beam

Development length

The development length Ld is defined as the length of embedment necessary to develop the full tensile strength of the bar.

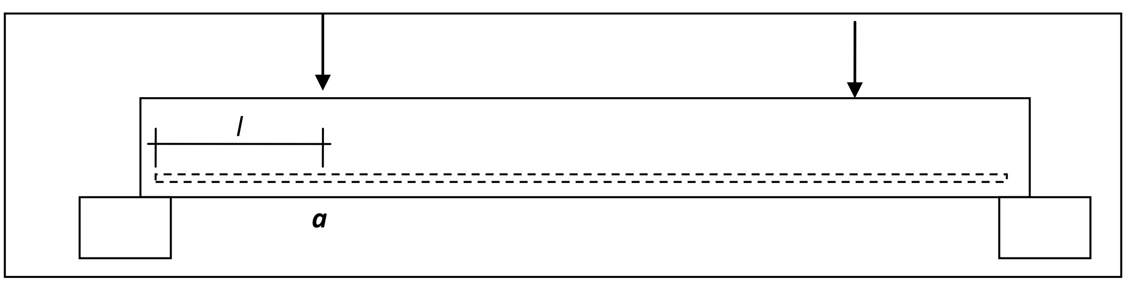


Figure 13.7 Development length

With reference to Figure 13.7 the moment, and therefore the steel stress, is maximum at point a, then the total tension force Asfs must be transferred from the bar to the concrete in the distance l by bond forces. To fully develop the strength of the bar the distance l must be equal to the development length Ld.

Formulae from the Mexican standard for the development length (NTC-2004)

The development length will be the one from multiplying the basic development length by the factor from the Table 5.1 from NTC-2004 page 130.

<u>Basic development length</u>

$$L_{db} = \frac{a_s f_y}{3\left(c + K_{tr}\right)\sqrt{f'_c}} \geq 0.11 \frac{d_b f_y}{\sqrt{f'_c}}$$

Where:

as= Transversal area of the steel bar

db = Bar diameter

c = spacing or concrete cover; one must use the smallest of the following:

Distance from the center bar to the nearest surface of concrete

Half the distance between center bars

Ktr = Index of transverse reinforcement = $\frac{A_{tr} f_{yv}}{100sn}$

Atr = total area of transverse bars inside the spacing s.

fyv = yielding stress of transverse steel

s = maximum spacing of transverse steel within a Ld distance, and

n = number or bars in the potential cracking plane

It is allowed to supouse Ktr = 0.

Ld never will be less than 300 mm.

Standard hooks

When there is not enough space for the steel to develop it yielding stress it is useful to use hooked bars.

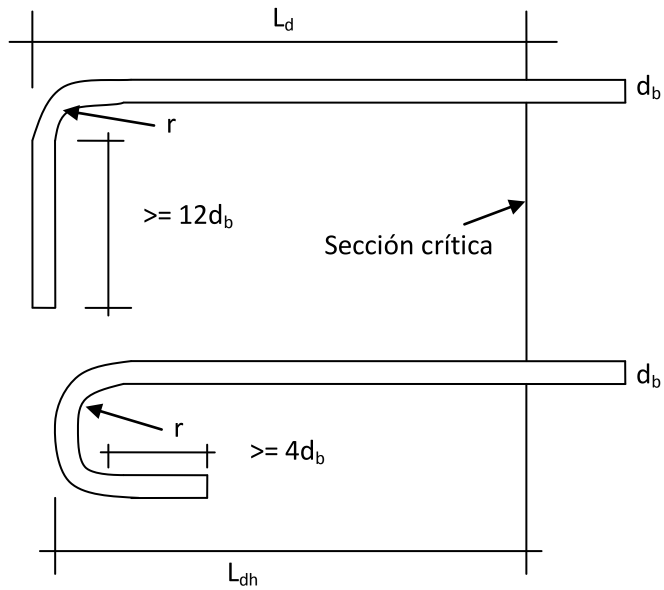


Figure 13.8 Development length for hooked bars (NTC-2004).

The radius for the bend point will never be less than:

$$r \geq \frac{f_y}{60\sqrt{f'_c}}$$

Or at least that the bar would be bent around a larger diameter bar.

The development length for hooked bars will be the one found by multiplaying the basic development length:

$$L_{dh} = \frac{0.076 d_b f_y}{\sqrt{f'_c}}$$

By the factor of the Table 5.2 from NTC-2004

Ldh will be never less than 150 mm or 8db.

Example 14

Calculate the development length for the hooked bar of the figure which is resisting the negative moment.

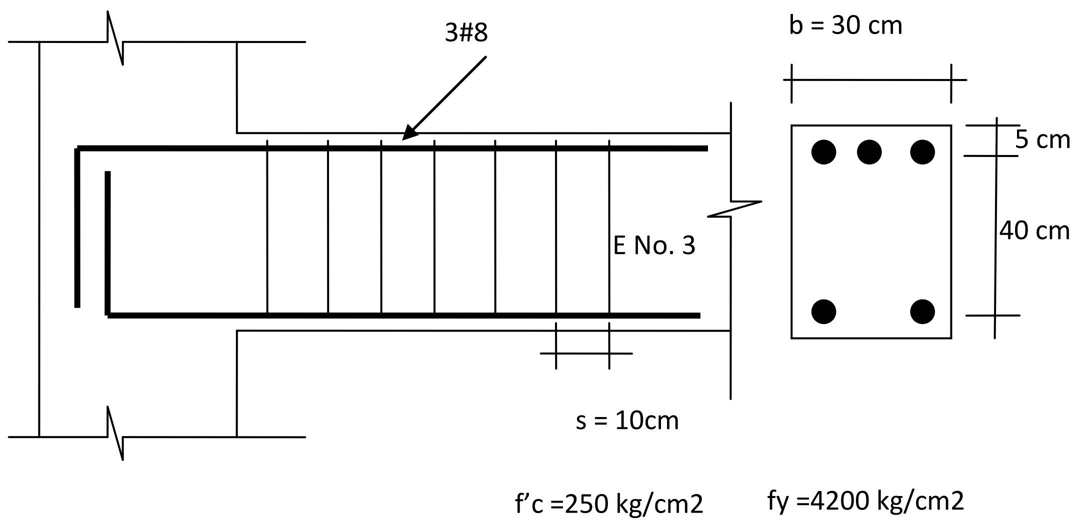


Figure 13.9

General Procedure of the solution:

First step, to compute the basic development length

Second step, to compute the development length

Formulae

$$L_{db} = \frac{a_s f_y}{3(c + K_y)\sqrt{f'_c}} \geq 0.11\frac{d_b f_y}{\sqrt{f'_c}} \qquad k_{tr} = \frac{A_{tr} f_w}{100sn} \qquad L_{dhook} = \frac{0.076 d_b f_y}{\sqrt{f'_c}}$$

Solution:

STEP 1

Development length for Noon hooked bar

$$L_{db} = \frac{a_s f_y}{3\left(c + K_y\right)\sqrt{f'_c}} \geq 0.11\frac{d_b f_y}{\sqrt{f'_c}}$$

But k_y= o

So:

$$L_{db} = \frac{a_s f_y}{3c\sqrt{f'_c}} \geq 0.11\frac{d_b f_y}{\sqrt{f'_c}}$$

$$L_{db} = \frac{(5 * 4200)}{(3 * 5)\sqrt{250}} = 88.53$$

Computing

$$k_{tr} = \frac{A_{tr} f_w}{100sn}$$

$$k_{tr} = \frac{(2 * 0.71)(4200)}{100(10 * 30)} = 1.99$$

$$L_{db} = \frac{a_s f_y}{3\left(c + K_y\right)\sqrt{f'_c}}$$

$$L_{db} = \frac{(5 * 4200)}{3(5 + 1.99)\sqrt{250}} = 63.35\ cm$$

Restriction

$$0.11\frac{d_b f_y}{\sqrt{f'_c}}$$

$$0.11\frac{(2.54 * 4200)}{\sqrt{250}} = 74.21\ cm$$

So, we need to use 74.21 cm. And we have to possible results.

STEP 2

Factor of the table 5.2 from NTC-2004

$$factor = (1.5 * 1.2 * 1.0) = 1.8$$

$$L_{ktr0} = (88.53 * 1.8) = 159.354\ cm$$

$$L_{ktr1.99} = (74.21 * 1.8) = 133.57\ cm$$

Development length for Hook´s bars

STEP 1

$$L_{dhook} = \frac{0.076 d_b f_y}{\sqrt{f'_c}}$$

$$L_{dh} = \frac{0.076(2.54 * 4200)}{\sqrt{250}} = 51.27\ cm$$

Restriction L_{dh} >15 cm or 8db

So we need to use L_{dh}= 51.27

STEP 2

Factor of the table 5.2 from NTC-2004

$$factor = (0.7 * 1.0) = 0.7$$

$$L_{dh} = (51.27 * 0.7) = 35.88\ cm$$

The hooks

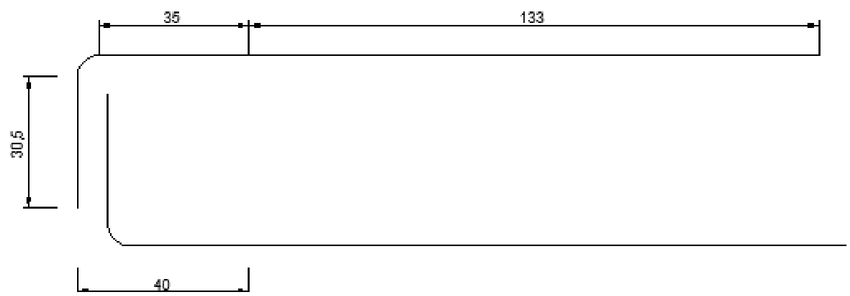


Figure 13.10

$$12d_b = (12 * 2.54) = 30.5\ cm$$

$$4d_b = (4 * 2.54) = 10.1\ cm$$

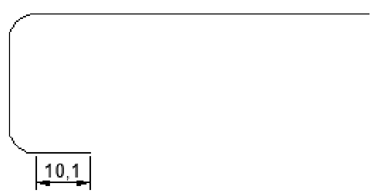


Figure 13.11

See Problem XIII from the annex, page 234

Development length for bars in compression (NTC-2004)

The development length for bars in compression will be at least 60% of the one require in tension without considering bent bars.

It will never be less than 200 mm.

Bar cut off and bend points in beams

The tensile force to be resisted by the reinforcement at any section is:

$$T = A_s f_y = \frac{M}{z}$$

Where M is the value of the bending moment at that section and z is the internal lever arm of the resisting moment.

Since it is desirable to design so that the steel everywhere in the beam is as nearly fully stressed as possible, it follows that the required steel area is nearly proportional to the bending moment.

From the expression

$$T = A_s f_y = \frac{M}{z}$$

It is clearly how the steel area is proportional to the bending moment.

Figure 14.1 shows, in percent, the steel requirement and the discontinued steel area.

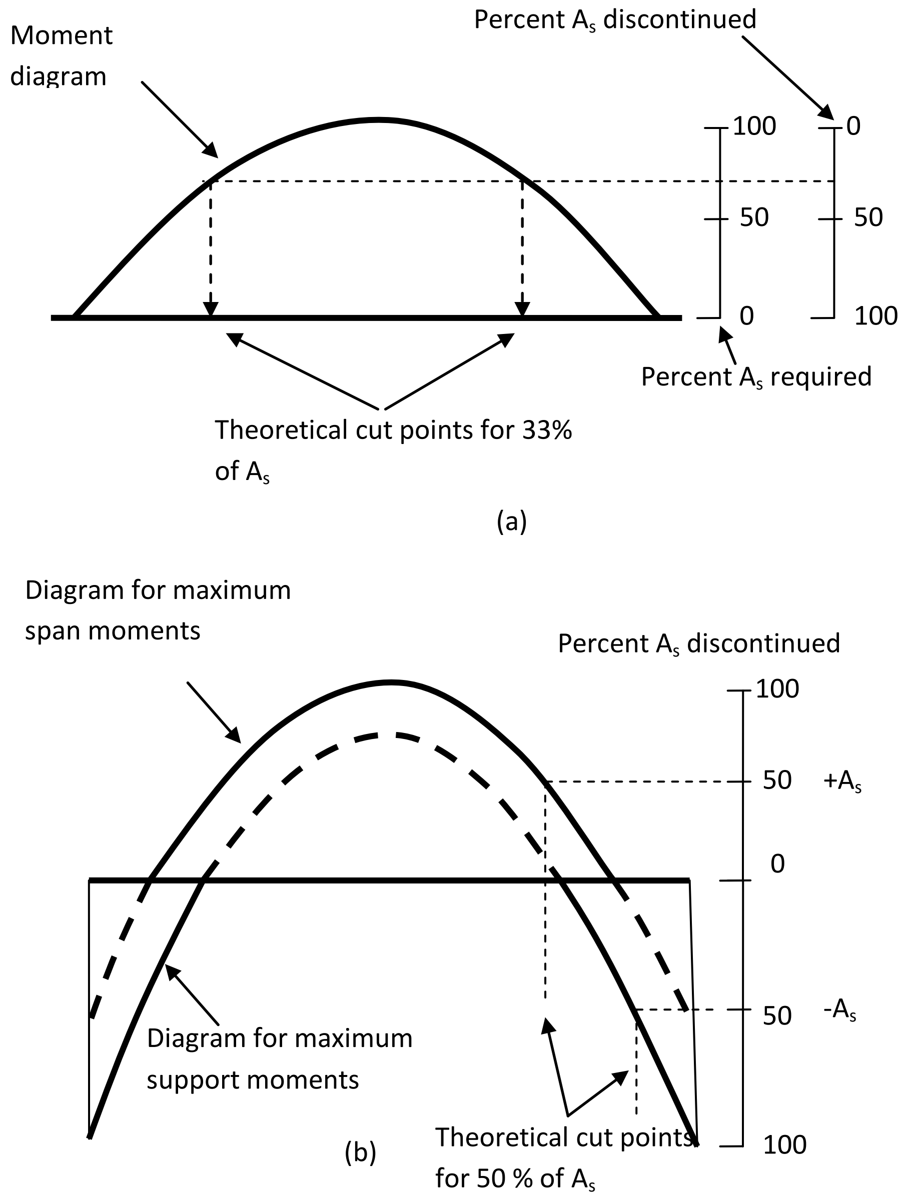


Figure 14.1 Bar cutoff points from moment diagrams. (a) Simple supported beam (b) Continuous beam (Nilson).

It is clear that the percentage of steel to cut, so that the remaining steel works at nearly fy, can be obtained from the moment diagram drawn at scale.

Restrictions

Steel reinforcement must not be cut at the section where is is not longer required according from the moment diagram.

Two are the main reasons:

- The uncertainty on the actual distribution of loads and,
- The approximate solutions in the structural analysis.

For these reasons all standards consider the actual cutoff points a distance beyon the theoretical cutoff points (NTC-2004).

The steel that is not longer necessary for flexion is cut or bent at a distance not less than the effective height beyond the theoretical cutoff point.

For the steel that is not cut or bent the distance to the next cuting point must be larger or equal to Ld + d.

For each side at a section of maximum moment the longitude of each bar must be larger or equal to the development length, Ld.

Complementary requirements

At ends of a simple supported beam a third part of total steel must be prolonged beyond the central part of the support, for continued beams the quantity of steel must be a quarter.

Figure 14.2 summarizes the latter points (NTC-2004)

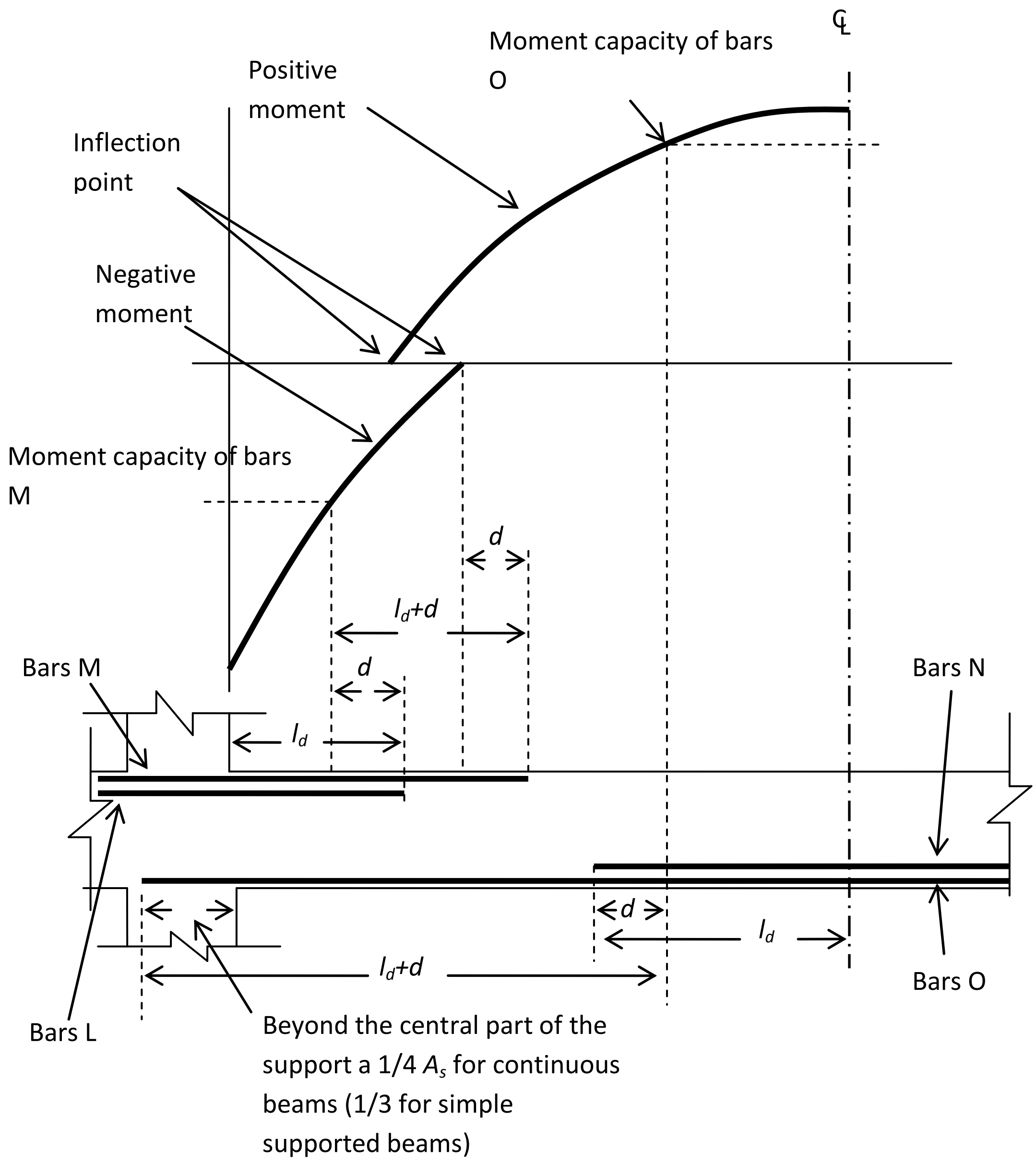


Figure 14.2 Cutoff points NTC-2004

Example 15

Perform steel cutting for the following beam

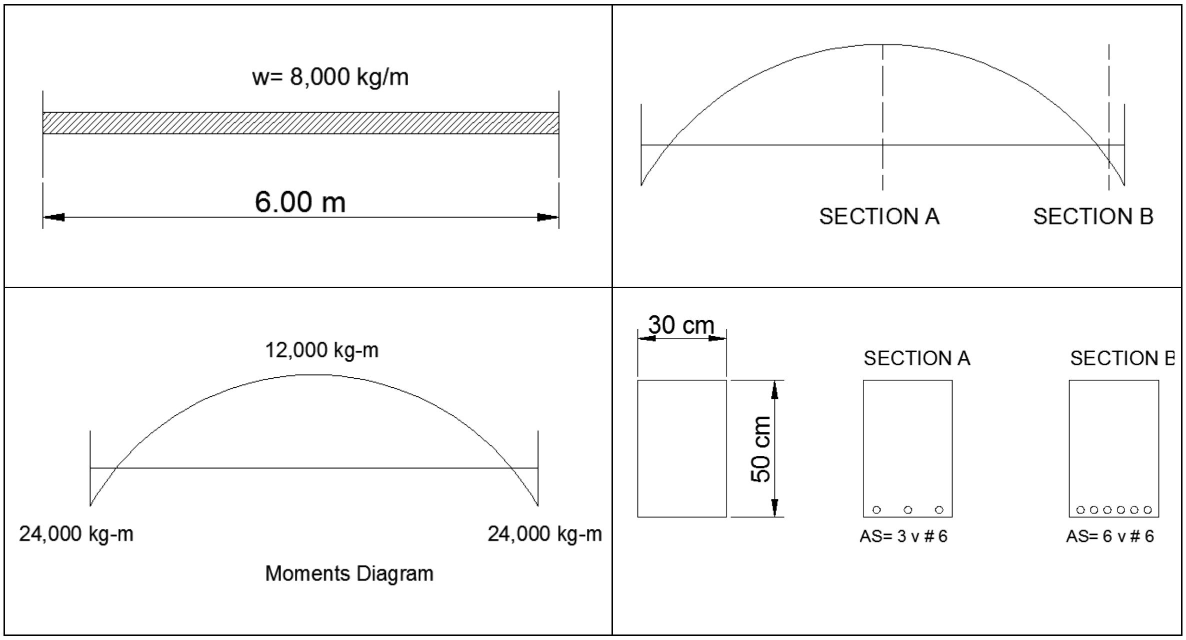


Figure 14.3

General Procedure of the solution:

The First step is to compute the development length

The Second step is to compute the first and the second condition

The Third step is to compute the first and the second condition with the theoretical point of cutting.

Formulae

$L_d = \dfrac{a_s f_y}{3c\sqrt{f'_c}}$	$0.11\dfrac{d_b f_y}{\sqrt{f'_c}}$	MR_{bars}

$T.P.C = theoritical\ point\ of\ cutting$ (from the moments SAP)

Solution:

STEP 1

Let use compute the development length.

Factor of the table 5.2 from NTC-2004

$$factor = (1.5 * 1.2 * 1.0) = 1.8$$

$$L_{d6} = (50.46 * 1.8) = 90.8\ cm$$

STEP 2

Cutting on section A

First condition

Let us compute L_d for a bar of number 6

$$L_{db} = \frac{a_s f_y}{3\left(c + K_{tr}\right)\sqrt{f'_c}} \geq 0.11 \frac{d_b f_y}{\sqrt{f'_c}}$$

$$L_{db} = \frac{a_s f_y}{3(c + k_{tr})\sqrt{f'_c}}$$

$$L_{db} = \frac{(2.85)(4{,}200)}{(3 * 5)\left(\sqrt{250}\right)}$$

$$L_{db} = 50.46\ cm$$

Remember that k_y= o, that's the reason for the past formulae

Let use compute the restriction

$$0.11\frac{d_b f_y}{\sqrt{f'_c}}$$

$$0.11\frac{(1.4 * 4200)}{\sqrt{250}} = 40.9\ cm$$

So we take the basic development length as the restriction is smaller.

Second condition

$$T.P.C + d$$

Let use compute the resistant moment for the section A with only two number six bars

$$MR_{2V\#6} = 10{,}166.9\ kg - m$$

From the moment diagram drawn at scale we find the theoretical point of cutting.

$$T.P.C = 0.65m$$

$$d = 0.50\ m$$

$$T.P.C + d = 1.15\ m = 115\ cm$$

As the second condition is larger than the first we take 115 cm as the distance for the cutting.

Cutting on section B

STEP 2

First condition

In this case the first condition is equal than for the cutting on section A as the bar is also number six bar.

Second condition

Cutting two bars

Let use compute the resistant moment for the section B with only four number six bars

$$MR_{4V\#6} = 19{,}120\ kg - m$$

From the moment diagram drawn at scale we find the theoretical point of cutting.

$$T.P.C(1) = 0.21m$$

$$d = 0.50\ m$$

$$T.P.C + d = 0.71\ m = 71\ cm$$

In this case the development length is larger than the second condition so the cutting distance is 90.8 cm

Cutting two more bars

Let use compute the resistant moment for the section B with only two number six bars

$$MR_{2V\#6} = 10{,}166.9\ kg - m$$

STEP 3

From the moment diagram drawn at scale we find the theoretical point of cutting.

$$T.P.C(2) = 0.66m$$

First condition

$$T.P.C\ (1) + l_d + d$$

$$= 0.21 + 0.9028 + 0.50 = 1.618m$$

$$= 161.8\ cm$$

Second condition

$$T.P.C\ (2) + d$$

$$0.66 + 0.50 = 1.16\ m$$

$$= 116\ cm$$

In this case the first condition is larger than the second condition so the cutting distance is 162 cm

Diagram for the bars

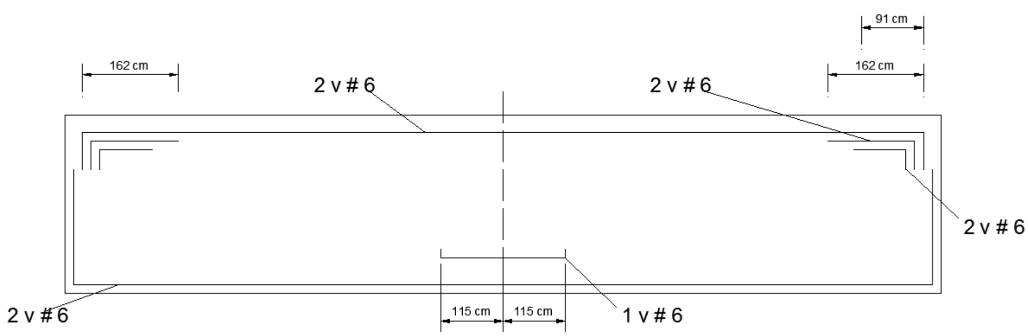


Figure 14.4

See Problem XIV from the annex, page 235

Chapter V

Analysis and Design of Slabs

V. Analysis and Design of Slabs

SYNOPSIS

Speaking of reinforcement concrete construction, the slabs are used to make surfaces thinner. A reinforced slab it's a flat plate, whit top and bottom surfaces. Sometimes it may be supported by reinforced concrete beams.

There are two types of slab:

- One-way slab
- Two-way edge supported slab

The flat slab system is one of most used beams methods of construction, the slab lays directly on the column and the load from the slab is directly transferred to the columns, if you want to support heavy load with slabs, you must increase the thickness, these are called drops or columns .

ANALYSIS AND DESIGN OF SLABS

In Reinforced concrete construction, slabs are used to provided flat, useful surfaces.

A reinforced concrete slab is a flat plate, usually horizontal, with top and bottom surfaces parallel or nearly so.

Slabs may be supported on two opposite sides only, in which case the structural action of the slab is essentially one-way.

There may be beams on all four sides, so that two-way slab action is obtained.

They may be supported directly on the ground and many other support configurations. Figure 15.1 shows some slab configuration.

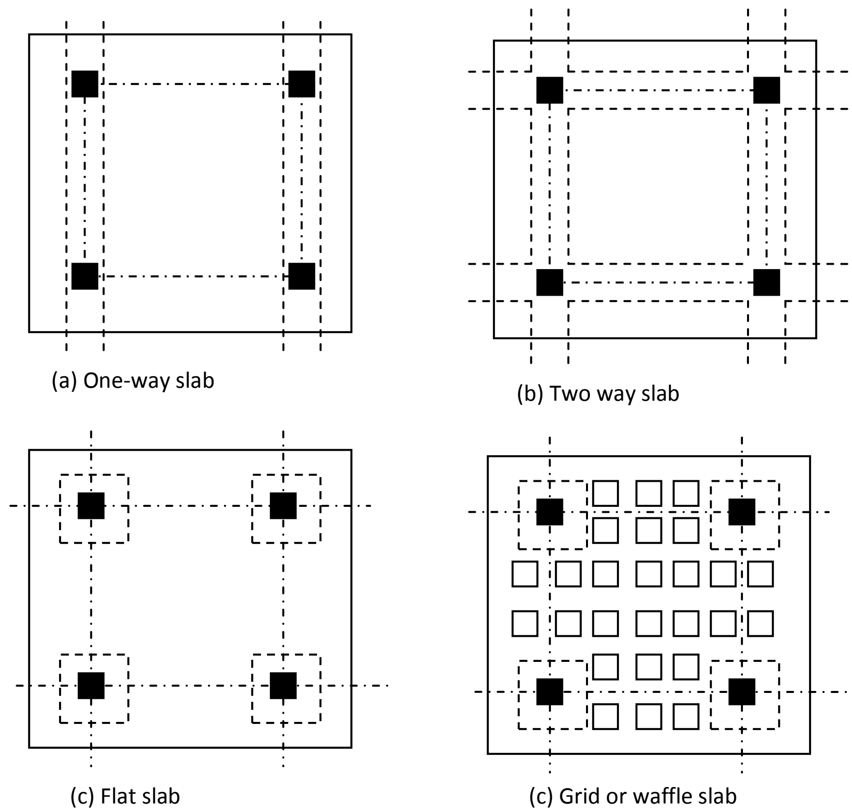


Figure 15.1 Types of structural slabs

<u>One way edge support slabs</u>

The structural action of a one-way slab may be visualized in terms of the deformed shape of the loaded surface. Figure 15.2 shows a rectangular slab, simply supported along its two opposite long edges and free of any support along the two opposite short edges.

If a uniformly distributed load is applied to the surface, the deflected shape will be as shown by the solid lines. Curvatures, and consequently bending moments, are the same in all strips spanning in the short direction between supported edges.

Whereas there is no curvature hence no bending moment, in the long direction

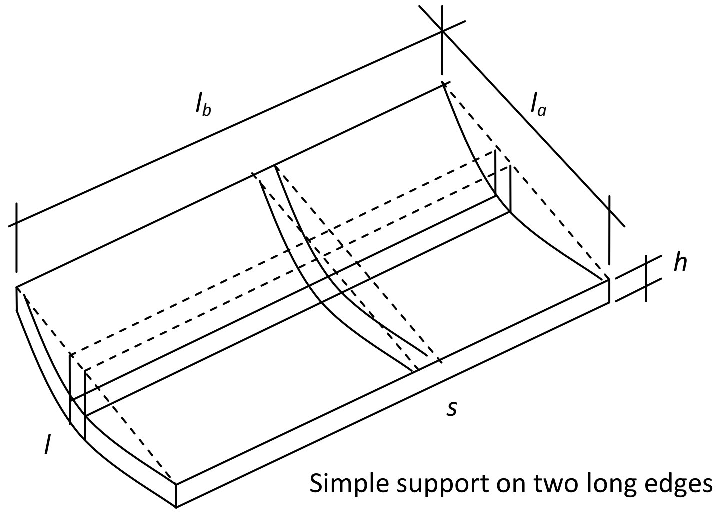


Figure 15.2. Deflected shape of uniformly loaded one-way slab.

For purposes of analysis and design, a unit strip of such slab cut out at right angles to the supporting beams may be considered as a rectangular beam of unit width, with a depth h equal to the thickness of the slab and span la equal to the distance between supported edges.

This strip can then be analyzed by the methods that were used for rectangular beams.

One-way slabs NTC-2004

In the design of one-way slabs the conditions for the design of rectangular beams are applicable.

Additional to the main reinforcement, one should provide temperature and shrinkage reinforcement.

Temperature and shrinkage reinforcement

In any direction in which the dimension of the structural element is larger than 1.5 m, the steel reinforcement will never be less than:

$$a_{s1} = \frac{660x_1}{fy(x_1 + 100)}$$

Where:

as1 = transversal steel area placed normal (at right angles) to the direction being considered by unit width (cm2/cm).

x1= minimal dimension measured at right angles to the main reinforcement (cm).

If x1 is not larger than 150 mm, the reinforcement can be placed in a single layer; if it is larger the reinforcement must be placed in two parallel layers near the element surfaces.

In elements exposed to the weather or in contact with the ground surface, the reinforcement will never be less than 1.5as1.

For simplicity, a reinforcement ratio of 0.002 can be used for elements not being exposed to the weather, and 0.003 for elements exposed to the weather or in contact with the ground.

Spacing for the bars will never exceed 500 mm and not either 3.5x1.

Example 16

Design the one-way slab being supported on its long edges. Consider that the slab was built integral with the support.

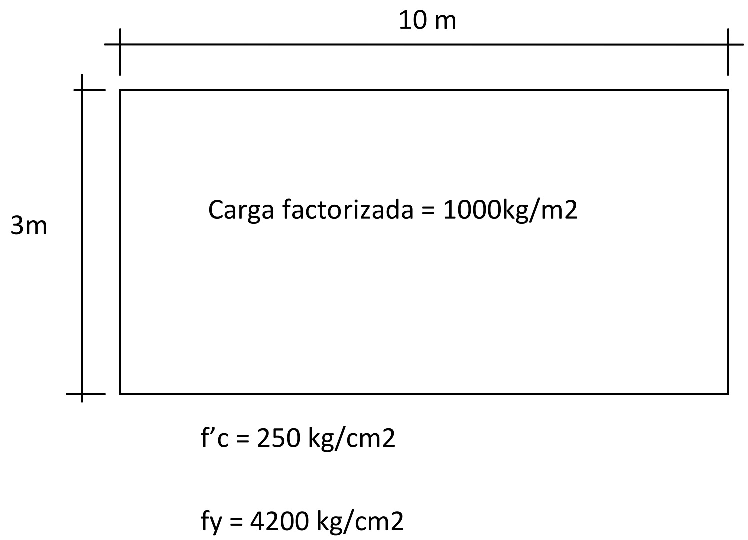


Figure 15.3

General procedure of solution:

The First step is to determine the total height of the slab

The Second step is to consider the reinforcement for the slab. In the principal direction by flexural analysis and in the other direction we will compute reinforcement by shrinkage a temperature effects.

Formulae

Height of slab	Restriction 1	
$h = \frac{p}{250}$	$fs = 0.60\, fy$	$factor = 0.032\sqrt[4]{(fs) * (w)}$
$distance = \frac{A_{No.3}}{a_{s1}}$	$a_{s1} = \frac{660\, x_1}{fy\,(x_1 + 100)}$	

Solution:

STEP 1

Computing the effective height of the slab perimeter

For computing the perimeter you need to consider the following: First, all non continuous sides must be multiply by a factor equal to 1.25 if the slab was not monolithically with its supports or 1.5 if the slab was not built monolithically with its supports. Second, if the slab is very large it is not necessary to consider a slab with the dimensions: long side equals to two times the short side.

$$p = 1.25\ (300 + 300) + 600 + 600$$

$$p = 1{,}950\ cm$$

Height of slab

$$h = \frac{1{,}950}{250} = 7.8\ cm$$

Restriction 1

$$fs = 0.60\ fy$$

$$fs = (0.60) * \ (4{,}200)$$

$$fs = 2{,}520\ kg/cm2$$

Restriction 2

W ≥ 380 Kg/m2

Computing w

$$w = 714 + (0.078) * (2{,}400)$$

$$w = 901.2\ kg/cm2$$

If the restrictions are not accomplishing so you need to compute a correction factor for the total height of the slab.

Computing the factor of amplification for “h”

$$factor = 0.032\sqrt[4]{(2520) * (901.2)}$$

$$factor = 1.24$$

Final height

$$h_f = (1.24 * 7.8) = 9.68\ cm$$

$$h_f = 10.0\ cm$$

Notice that as we increase the final height of the slab we need to re-compute the final load of the slab.

STEP 2

Final load

$$w_f = 714 + (0.1 * 2{,}400)$$

$$w_f = 954 kg/m2$$

$$w_u = (1.4 * 954)$$

$$w_u = 1335\ kg/m2$$

With the final load we compute the final moments so we can compute the reinforcement, for the short direction.

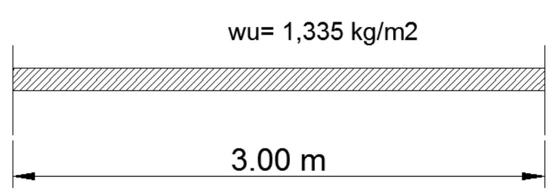


Figure 15.10

The unit strip section

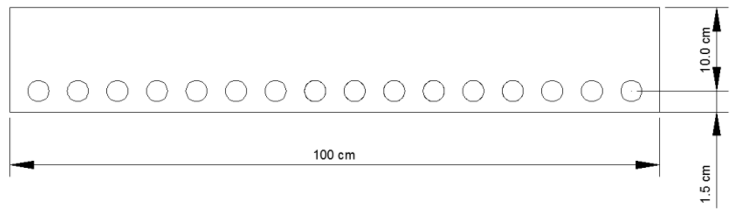


Figure 15.11

Moments

$$w_s = \frac{wl^2}{12}$$

$$w_s = \frac{(1335)(3^2)}{12} = 1{,}001\ kg - m$$

$$w_m = \frac{wl^2}{24}$$

$$w_s = \frac{(1335)(3^2)}{24} = 500.5\ kg - m$$

Diagram

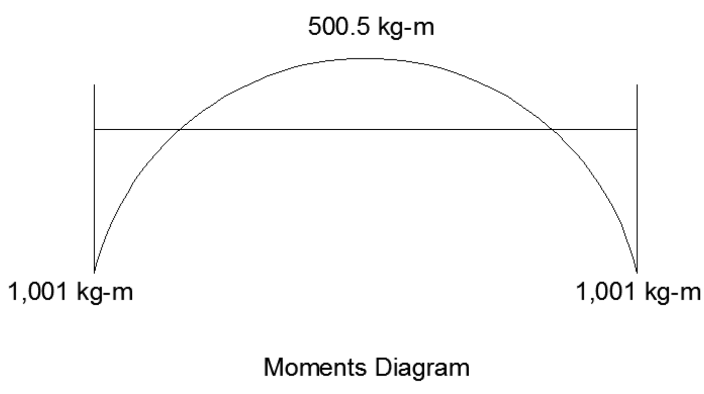


Figure 15.12

For negative moment

No. 3 bars @ 25 cm (for the minimum requirements)

For positive moment

No. 3 bars @ 25 cm (for the minimum requirements)

Restriction

$$3.5xi = 3.5 * (11.5) = 40.25\ cm$$

No. 3 bars @ 40 cm

Temperature and shrinkage reinforcement (long direction)

$$a_{s1} = \frac{660\ x_1}{fy\ (x_1 + 100)}$$

$$a_{s1} = \frac{(660 * 11.5)}{(4{,}200 * (11.5 + 100))}$$

$$a_{s1} = 0.016\ cm2/cm$$

Using bars of No. 3 let us compute the distance

$$distance = \frac{A_{No.3}}{a_{s1}}$$

$$distance = \frac{0.71}{0.016} = 44\ cm\ \approx 40\ cm$$

Figure 2, reinforcement in the slab.

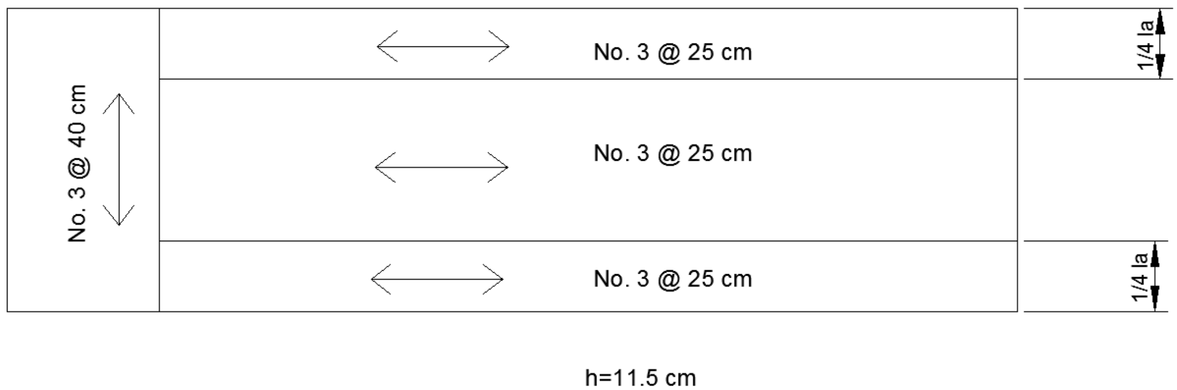


Figure 15.13

See Problem XV from the annex, page 236

<u>Two-way slabs edge support slabs</u>

If the ratio length to width is equal or larger than two, most of the load is resisted in the short direction so that the slab can be considered as a one-way slab.

In many cases, however, rectangular slabs bent into a dished surface rather than a cylindrical one. That means that at any point the slab is curved in both principal directions, and since bending moments are proportional to curvatures, moments also exist in both directions.

To resist these moments, the slab must be reinforced in both directions, by at least two layers of bars perpendicular to each other.

The simplest type of two-way slab action is that represented in Figure 15.14. To visualize its flexural performance, it is convenient to think of it as consisting of two sets of parallel strips, in each one of two directions, intersecting each other.

Evidently, part of the load is carried by one set and the remainder by the other.

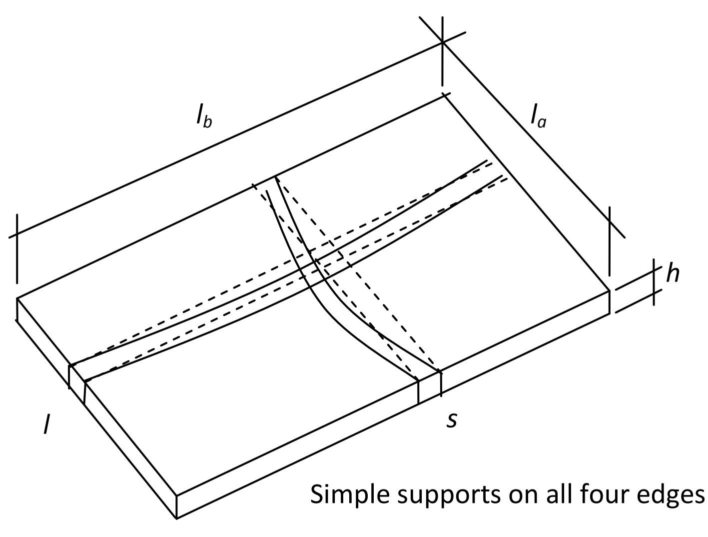


Figure 15.14 Two-way slab

Figure 15.14 shows the two center strips of a rectangular plate with short span la and a long span lb. If the uniform load is w, each strip acts approximately like a simple beam, uniformly loaded by its share of w.

Because these imaginary strips actually are part of the same monolithic slab, their deflections at the intersection are the same.

Equating the center deflections of the short and long strips

$$\frac{5w_a l_a^4}{384EI} = \frac{5w_b l_b^4}{384EI}$$

$$\frac{w_a}{w_b} = \frac{l_b^4}{l_a^4}$$

Where wa is the share of load w carried in the short direction and wb is the share of load w carried in the long direction.

Considering that wa + wb = w it follows:

$$w_a = \frac{wl_b}{l_a + l_b} \text{ ; y}$$

$$w_b = \frac{wl_a}{l_a + l_b}$$

One sees that the larger share of the total load is carried in the short direction.

This result is approximated because the actual behavior of a two-way slab is more complex than that of the two intersecting strip

An understanding of the behavior of a slab is gained from Figure 15.15 which shows a slab model consisting of two sets of three strips each.

Consider the outer strips s2 and l2, however, area not only bent but twisted. It is seen that the element at the intersection is not uniformly deformed, so it is conclude that torsion exists in the element.

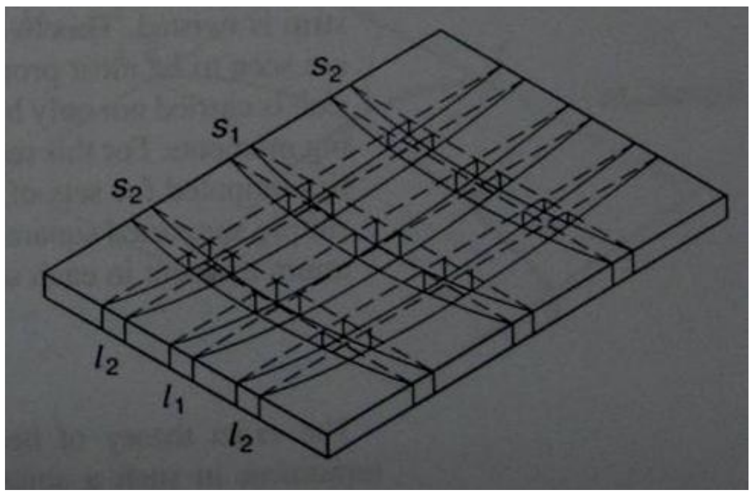


Figure 15.15. Two-way slab (grid model)

So the load w is not only relisted by bending moments but also by torsion moments. So the actual bending moments are smaller than the computed from the last procedure.

On the other hand, it is well known that the largest bending moment occurs at the center of the short span.

If the strip were an isolated beam and the load were increased such a beam would fail at the moment that the steel reach its yielding point. However, considering the slab as a whole, one sees that no immediate failure occurs.

The neighboring strips, being actually monolithic with it, will take over any additional load that the strip can no longer carry until they, in turn, start yielding.

This inelastic redistribution will continue until a large area in the central part of the slab all the steel in both directions is yielding. Only then the entire slab will fail.

Consider the following example:
In a square slab wa = wb = w/2

If only bending moment were present the moment in the central strip would be:

((w/2)l2)/8 =0.0625wl2

As torsion moments are present the last value is reduce:

0.048wl2

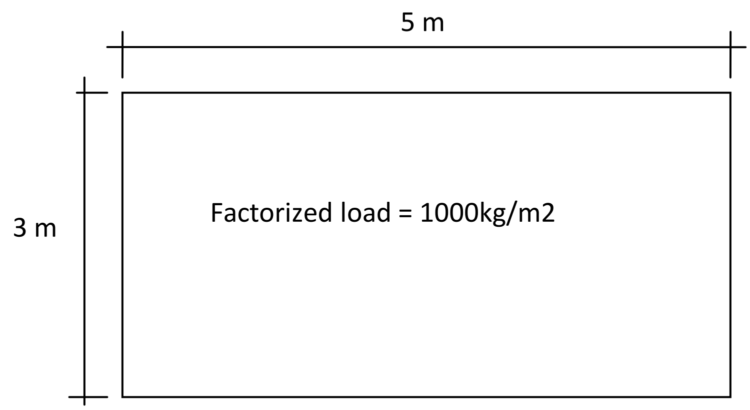
As the load works as a monolithic structure, the value is once more reduce:

0.036wl2

Note that a 57% reduction is found.

Example 17

Design the two-way slab being supported on its four edges.



f'c = 250 kg/cm2

fy = 4200 kg/cm2

Figure 15.15

General procedure of the solution:

The First step is to check if the slab is one-way or two-way.

The Second step is considering the reinforcement for the slab. In the two way-slabs, analyze in the long and short direction.

Formulae

$$\frac{L_a}{L_b} > 2 \qquad h = \frac{p}{250} \qquad fs = 0.60\,fy \qquad Live\ Load = \frac{1000}{1.4}$$

$$Dead\ Load = (0.096)x(fy) \qquad w_u = \frac{w * l_a}{L_a + L_b} \qquad w_u = \frac{w * l_b}{L_a + L_b}$$

Solution:

STEP 1

Restriction

$$\frac{L_a}{L_b} > 2$$

For the slab

$$\frac{5}{3} = 1.66$$

So it is two-way slab

STEP 2

Compute the Perimeter: considering that the slab was not build monolithically with its supports

For computing the perimeter you need to consider the following: First, all non continuous sides must be multiply by a factor equal to 1.25 if the slab was not monolithically with its supports or 1.5 id the slab was not built monolithically with its supports. Second, if the slab is very large it is not necessary to consider a slab with the dimensions: long side equals to two times the short side.

$$p = 1.5\ (300 + 300 + 500 + 500)$$

$$p = 2{,}400\ cm$$

Height of slab

$$h = \frac{2{,}400}{250} = 9.6\ cm$$

Restriction

$$fs = 0.60\ fy$$

$$fs = (0.60) * \ (4{,}200)$$

$$fs = 2{,}520\ kg/cm2$$

Let us compute the service load “w”

$$Live\ Load = \frac{1000}{1.4}$$

$$Live\ Load = 714\ kg$$

$$Dead\ Load = (0.096)x(4{,}200)$$

$$Dead\ Load = 230.4\ kg$$

$$Wservice = 714 + 230$$

$$Wservice = 944\ kg$$

Restriction

$$Wservice \geq 380\ kg/m2$$

If the restrictions are not accomplishing so you need to compute a correction factor for the total height of the slab.

Computing the factor of amplification for “h”

$$factor = 0.032\sqrt[4]{(f_s * w)}$$

$$factor = 0.032\sqrt[4]{(2520) * (944)}$$

$$factor = 1.25$$

Final height

$$h_f = (1.25 * 9.6) = 12\ cm$$

$$h_f = 12.0\ cm$$

Let us compute the total load

$$DL = (0.12 * 2400)$$

$$DL = 288\ kg/m2$$

Total service load

$$Wservice = 714 + 288$$

$$Wservice = 1{,}002\ kg/m2$$

Ultimate load

$$Wu = 1.4 * Wservice$$

$$Wu = (1.4 * 1{,}002)$$

$$Wu = 1{,}402\ kg/m2$$

Short direction

$$w_u = \frac{w * l_b}{L_a + L_b}$$

$$w_u = \frac{(1{,}402 * 5)}{3 + 5}$$

$$w_u = 876\ kg/m$$

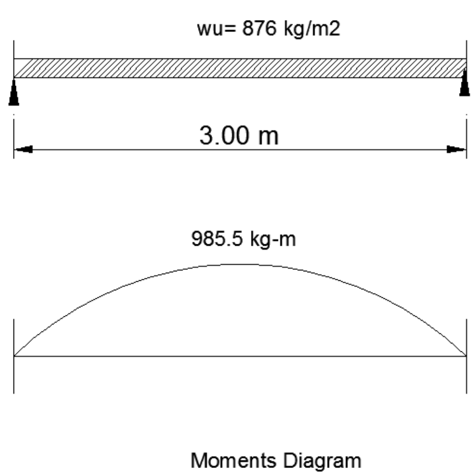


Figure 15.16

Long direction

$$w_u = \frac{w * l_a}{L_a + L_b}$$

$$w_u = \frac{(1{,}402 * 3)}{3 + 5}$$

$$w_u = 526\ kg/m$$

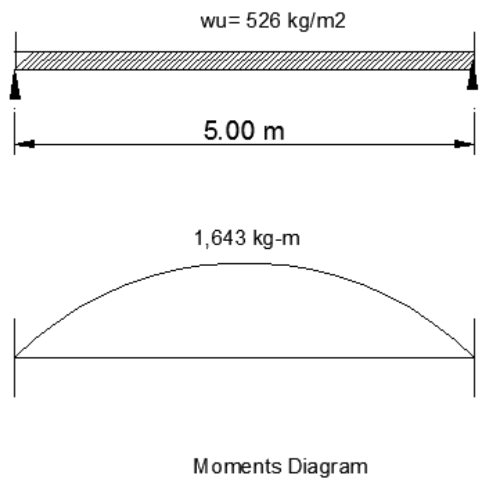


Figure 15.17

Notice than the bending moment for the long side is larger than for the short side, this result does not make sense as we know that the moment in the short side must be biggest of the two moments.

These results are approximated because the actual behavior of two-way slab is more complex than that of the two intersecting strips. In this approach only the bending moments are taken into account. However, in a two way slab bending and torsion moments are present plus the monolithic nature of the slab

See Problem XVI from the annex, page 237

Factor method (NTC-2004)

Bending moments due to uniformly distributed loads

Bending moments in slabs supported along its perimeter will be computed form the coefficients on table 6.1 while they accomplish the following limitations:

Slabs are almost rectangular;

Distributed loads are approximately uniformly in each slab

Negative moments in a common support are not different in a quantity of 50% of the smallest of them; and

The ratio live load to dead load is not larger than 2.5 for slab monolithic with its supports and 1.5 for other cases.

For intermediate values of the ratio m, (short span a1 to long span a2) interpolation will apply.

Table 15.18 Bending moment coefficients for rectangular panels, medians

Tabla 6.1 Coeficientes de momentos flexionantes para tableros rectangulares, franjas centrales[1]

Tablero	Momento	Claro	Relación de lados corto a largo, $m = a_1/a_2$													
			0		0.5		0.6		0.7		0.8		0.9		1.0	
			I[2]	II[3]	I	II	I	II	I	II	I	II	I	II	I	II
Interior Todos los bordes continuos	Neg. en bordes interiores	corto	998	1018	553	565	489	498	432	438	381	387	333	338	288	292
		largo	516	544	409	431	391	412	371	388	347	361	320	330	288	292
	Positivo	corto	630	668	312	322	268	276	228	236	192	199	158	164	126	130
		largo	175	181	139	144	134	139	130	135	128	133	127	131	126	130
De borde Un lado corto discontinuo	Neg. en bordes interiores	corto	998	1018	568	594	506	533	451	478	403	431	357	388	315	346
		largo	516	544	409	431	391	412	372	392	350	369	326	341	297	311
	Neg. en bordes dis.	largo	326	0	258	0	248	0	236	0	222	0	206	0	190	0
	Positivo	corto	630	668	329	356	292	306	240	261	202	219	167	181	133	144
		largo	179	187	142	149	137	143	133	140	131	137	129	136	129	135
De borde Un lado largo discontinuo	Neg. en bordes interiores	corto	1060	1143	583	624	514	548	453	481	397	420	346	364	297	311
		largo	587	687	465	545	442	513	411	470	379	426	347	384	315	346
	Neg. en bordes dis.	corto	651	0	362	0	321	0	283	0	250	0	219	0	190	0
	Positivo	corto	751	912	334	366	285	312	241	263	202	218	164	175	129	135
		largo	185	200	147	158	142	153	138	149	135	146	134	145	133	144
De esquina Dos lados adyacentes discontinuos	Neg. en bordes interiores	corto	1060	1143	598	653	530	582	471	520	419	464	371	412	324	364
		largo	600	713	475	564	455	541	429	506	394	457	360	410	324	364
	Neg. en borde discontinuos	corto	651	0	362	0	321	0	277	0	250	0	219	0	190	0
		largo	326	0	258	0	248	0	236	0	222	0	206	0	190	0
	Positivo	corto	751	912	358	416	306	354	259	298	216	247	176	199	137	153
		largo	191	212	152	168	146	163	142	158	140	156	138	154	137	153
Extremo Tres bordes discontinuos un lado largo continuo	Neg. en borde cont.	corto	1060	1143	970	1070	890	1010	810	940	730	870	650	790	570	710
	Neg. en bordes disc.	corto	651	0	370	0	340	0	310	0	280	0	250	0	220	0
		largo	220	0	220	0	220	0	220	0	220	0	220	0	220	0
	Positivo	corto	751	912	730	800	670	760	610	710	550	650	490	600	430	540
		largo	185	200	430	520	430	520	430	520	430	520	430	520	430	520
Extremo Tres bordes discontinuos un lado corto continuo	Neg. en borde cont.	largo	570	710	570	710	570	710	570	710	570	710	570	710	570	710
	Neg. en borde disc.	corto	570	0	480	0	420	0	370	0	310	0	270	0	220	0
		largo	330	0	220	0	220	0	220	0	220	0	220	0	220	0
	Positivo	corto	1100	1670	960	1060	840	950	730	850	620	740	540	660	430	520
		largo	200	250	430	540	430	540	430	540	430	540	430	540	430	540
Aislado Cuatro lados discontinuos	Neg. en bordes discontinuos	corto	570	0	550	0	530	0	470	0	430	0	380	0	330	0
		largo	330	0	330	0	330	0	330	0	330	0	330	0	330	0
	Positivo	corto	1100	1670	830	1380	800	1330	720	1190	640	1070	570	950	500	830
		largo	200	250	500	830	500	830	500	830	500	830	500	830	500	830

1 Para las franjas extremas multiplíquense los coeficientes por 0.60.

2 Caso I. Losa colada monolíticamente con sus apoyos.

3 Caso II. Losa no colada monolíticamente con sus apoyos.

Los coeficientes multiplicados por $10^{-4} w a_1^2$, dan momentos flexionantes por unidad de ancho; si w está en kN/m² (en kg/m²) y a_1 en m, el momento da en kN-m/m (en kg-m/m)

Para el caso I, a_1 y a_2 pueden tomarse como los claros libres entre paños de vigas; para el caso II se tomarán como los claros entre ejes, pero sin exceder del claro libre más dos veces el espesor de la losa.

Critical sections and strips for reinforcement

For negative moment the critical section will be taken at the edge of the slab, and for positive moments at the intermediate lines.

In order to place the reinforcement, the slab will be divided, in each direction, in two lateral strips and one intermediate.

For a ratio m, long span to short span, larger than 0.5, the intermediate strips will be half the span perpendicular to them, and the lateral strips will be a quarter.

For a ratio m, long span to short span, smaller than 0.5, the intermediate strips will be (a2–a1), and the lateral strips will be a1/2.

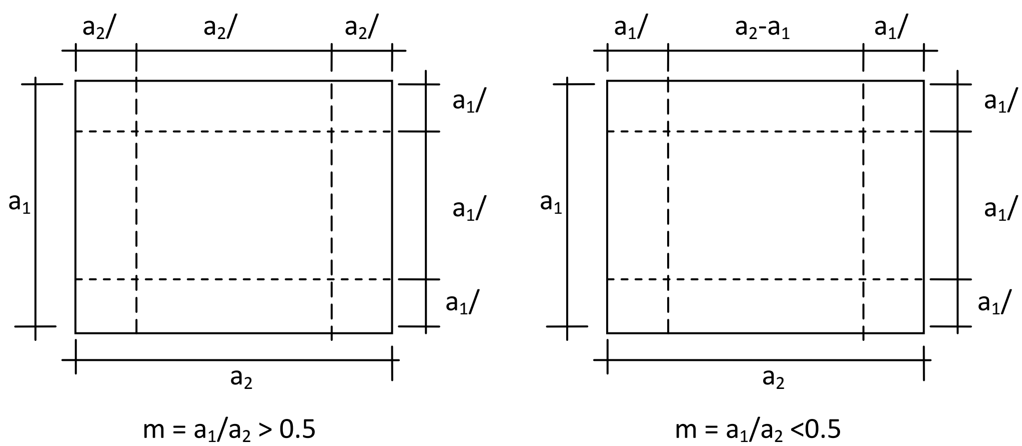


Figure 15.19 Width of strips according to m

In order to bend the reinforcement and to apply the requirements for anchorage inflexion lines will be suppose at one sixth of the short span from the edge of the slab for positive moment and one fifth of the short span from the edge of the span for negative moment.

<u>Distribution of moments between adjacent slabs</u>

When the moments at the edge of two adjacent slabs are different, two thirds of the difference will be distributed between the slabs if the slab is cast monolithically with the supports and the whole difference if the slab is not cast monolithically with the supports. For the distribution the rigidity of the slab will be proportional to $d^3/a1$.

Other disposition about the reinforcement

The dispositions about maximal spacing and reinforcement ratio will be applied (section 4.9, about minimal covers and 5.7 temperature and shrinkage reinforcement, respectively).

At the proximity of concentrate loads above 1000 kg, the spacing will be not larger than 2.5d, where d is the effective height of the slab.

<u>Minimum thickness</u>

When the table 6.1 can be applied, the computation of deflections can be omitted if the effective height of the slab is not smaller than the perimeter of the slab divided by 250 for type I concrete, and 170 for concrete type II.

For the computation the length of discontinuous sides will be multiplied by 1.50 if the slab is not cast monolithically with the supports and 1.25 if the slab is cast monolithically with the supports.

For long slabs it is not necessary to compute a thickness corresponding to a slab with the following relationship a2=2a1.

Last limitation is applicable for slabs in which

$$f_s \leq 2520 \text{ kg/cm}^2 \text{ y } w \leq 380 \text{ kg/m}^2$$

For other combinations of fs and w, the thickness will be multiply by the following factor:

$$0.032\sqrt[4]{f_s w}$$

For this expression fs is the stress in the steel for service conditions , in kg/cm² and w is the load in service conditions also, in kg/m², (fs can be suppose equal to 0.6fy).

Shear analysis

The critical section will be taken at an effective height from the edge. The shear force acting in a unit width of slab is:

$$V = \left(\frac{a_1}{2} - d\right)\left(0.95 - 0.5\frac{a_1}{a_2}\right)w$$

Or the one determined with a more precise analysis. If there are continuous and not continuous edges, V will be multiply by 1.15.

The resistance of the slab will be computed from:

$V_{Rlosa} = 0.5F_R bd\sqrt{f^*c}$ (The reinforcement is not taken into account).

Example 18

Design the two-way slab being supported on its four edges. a) Consider the slab being cast monolithically with its supports; b) consider the slab not being cast monolithically with its supports. Apply the coefficients form NTC-2004, table 6.1.

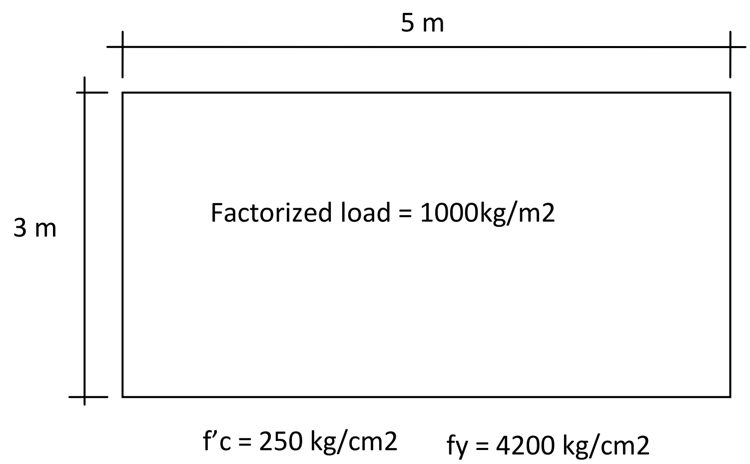


Figure 15.20

Formulae

$m = \dfrac{a_1}{a_2}$	$factor = \dfrac{w_o(a_1)^2}{1{,}000}$

General procedure of the solution:

The First step is to calculate the ratio.

The Second step is compute the final moments whit the coefficients from the table 6.1 page 412 from NTC-2004

The Third step is to make a graphical representation for the moments.

Note: *Due this for the two cases

Solution:

STEP 1

a) Consider the slab being cost monolithically with the supports.

Let us compute "m" ratio short to long sides

$$m = \frac{a_1}{a_2}$$

$$m = \frac{3}{5} = 0.6$$

As we have an isolate slab, we have coefficients for the third column of table 15.18 now let us compute the factor, so we can get the final moments on the slab.

$$factor = \frac{w_o(a_1)^2}{10{,}000}$$

$$factor = \frac{1{,}402(3)^2}{10{,}000} = 1.26$$

So we need to multiply the factor whit the coefficient to compute the moment (Column five on table 1)

$$M = 530 * 1.26 = 667.8$$

$$M = 330 * 1.26 = 415.8$$

$$M = 800 * 1.26 = 1008$$

$$M = 500 * 1.26 = 630$$

STEP 2

Table 1.- Computation of moments

		Coeficient	Moment
Negative on continuous edges	short	530	667.8
	long	330	415.8
Positive	short	800	1008
	long	500	630

STEP 3

Graphical representation of the moments

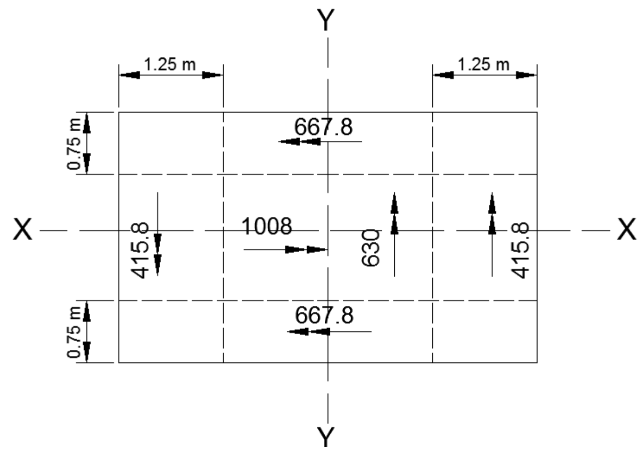


Figure 15.21

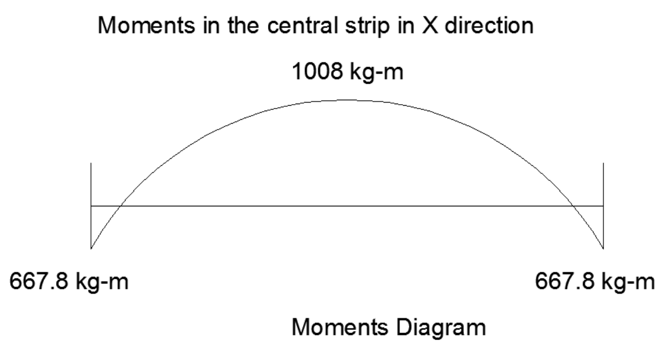


Figure 15.22

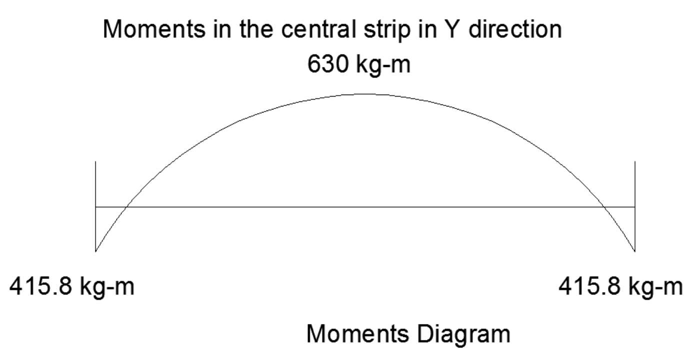


Figure 15.23

STEP 1

b) Consider the slab not being built monolithically whit it supports

We have an isolate slab, and we need to compute the factor.

$$factor = \frac{w_o(a_1)^2}{1{,}000}$$

$$factor = \frac{1{,}402(3)^2}{1{,}000} = 1.26$$

So we need to multiply the factor whit the coefficient to compute the moment

$$M = 0 * 1.26 = 0$$

$$M =* 1.26 = 0$$

$$M = 1330 * 1.26 = 1675.8$$

$$M = 830 * 1.26 = 1045.8$$

STEP 2

Table 2.- computation of the moments

		Coeficient	Moment
Negative on continuous edges	Short	0	667.8
	Long	0	415.8
Positive	Short	1330	1,675.80
	Long	830	1,045.80

STEP 3

Graphical representation of the moments

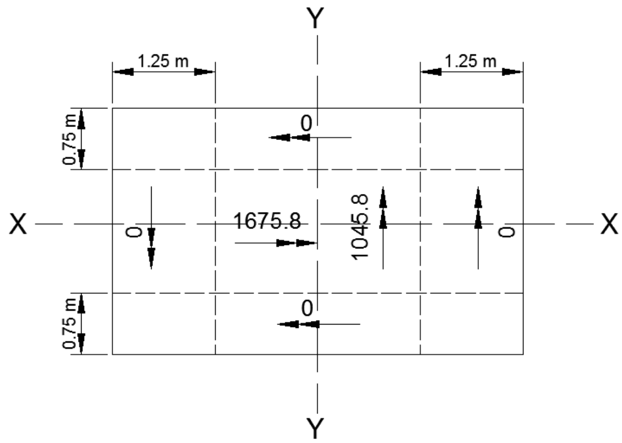


Figure 15.24

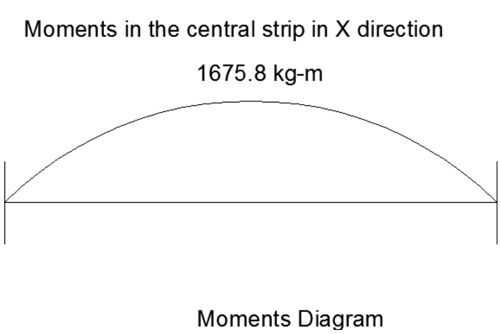


Figure 15.25

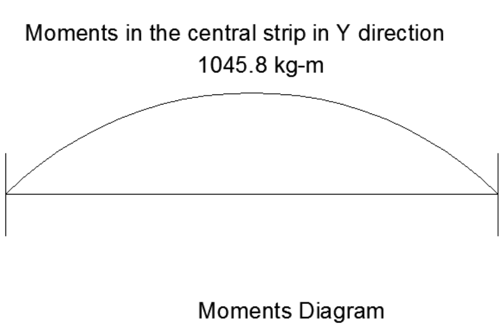


Figure 15.26

Shear analysis

$$V_u = \left(\frac{a_1}{2} - d\right)\left(0.45 - 0.5\,\frac{a_1}{a_2}\right)w$$

Let us consider a concrete cover of 1.5 cm

d=12-1.5= 10.5 cm

$$V_u = \left(\frac{3}{2} - 0.105\right)\left(0.45 - 0.5\,\frac{3}{5}\right)1402$$

$$V_u = 1{,}271\ kg$$

$$VR_{losa} = 0.5\ F_R BD\sqrt{F^*C}$$

$$VR_{losa} = (0.5)(0.8)(100)\sqrt{(0.8) * (250)}$$

$$VR_{losa} = 5{,}939.6\ KG$$

$$VR_{losa} > V_u.\,OK$$

See Problem XVII from the annex, page 238

Example 19:

Determine the reinforcement for the slab. Consider the slab being cast monolithically with its supports. Apply the coefficients from table 6.1.

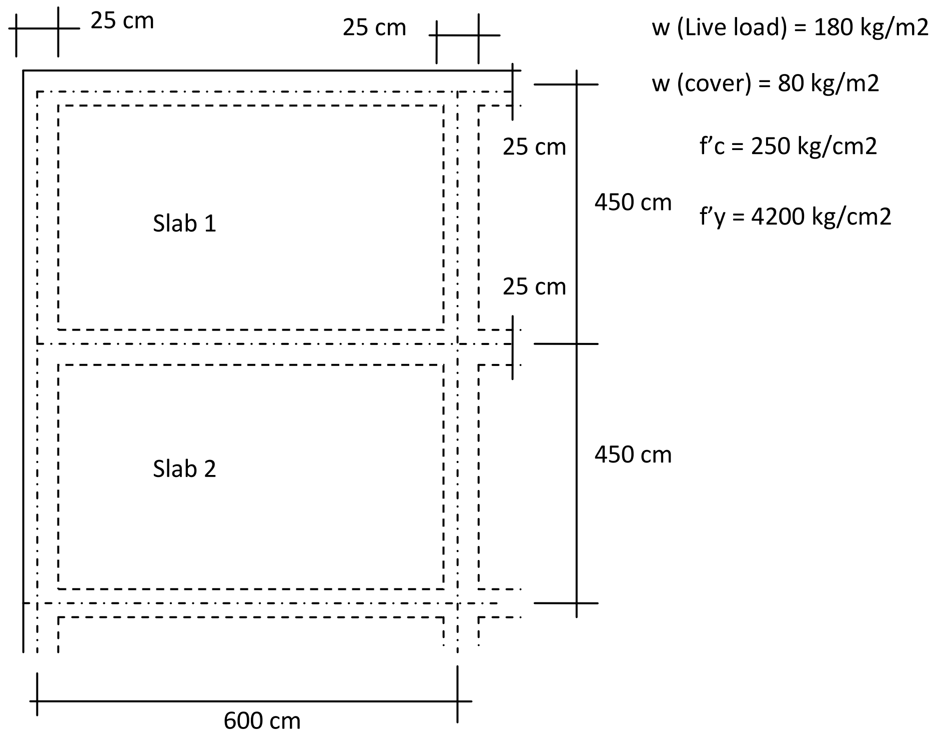


Figure 15.9

General procedure of the solution:

The First step is to determine the height slab

The Second step is to compute the total load

The Third step is to compute the moments

Fourth step compute the final moment

Formulae

$$factor = 0.032\sqrt[4]{(f_s * w)} \qquad m = \frac{a_1}{a_2} \qquad factor = \frac{w_o(a_1)^2}{10,000}$$

Solution:

STEP 1

From the corner slab which has the more discontinuous sides that will lead to a higher height

Perimeter:

$$p = 1.25\ (600 + 450) + (600 + 450)$$

$$p = 2{,}362\ cm$$

Height of slab

$$h = \frac{2{,}362}{250} = 9.51\ cm$$

Let us compute the dead load.

$$Dead\ Load = (0.095)x(4{,}200)$$

$$Dead\ Load = 228\ kg$$

$$Dead\ Load = 228 + 80 = 308$$

$$live\ load = 180\ kg/m2$$

$$Wservice = 308 + 180 = 488\ kg$$

Restrictions

$$Wservice < 380\ kg$$

$$fs \leq 2{,}520\ kg/cm2$$

As we do not accomplish last restriction we need to compute the correction.

Computing the factor of amplification for “h”

$$factor = 0.032\sqrt[4]{(f_s * w)}$$

$$factor = 0.032\sqrt[4]{(2520) * (488)}$$

$$factor = 1.06$$

Final height

$$h_f = (1.06 * 9.51) = 10.12cm$$

$$h_f = 11.0\ cm$$

STEP 2

Let us compute the total load

$$DL = (0.11 * 2400)$$

$$DL = 264\ kg/m2$$

$$DL = 264 + 80 = 344\ kg/cm2$$

Total service load

$$Wservice = 180 + 344$$

$$Wservice = 524\ kg/m2$$

Ultimate load

$$Wu = 1.4 * Wservice$$

$$Wu = (1.4 * 524)$$

$$Wu = 734\ kg/m2$$

STEP 3

Let us compute “moments”

$$m = \frac{a_1}{a_2}$$

$$m = \frac{450}{600} = 0.75$$

$$factor = \frac{w_o(a_1)^2}{10,000}$$

$$factor = \frac{734(4.5)}{10,000} = 1.48$$

corner		Coeficient	Moment
Negative on inside edges	Short	445	658.6
	Long	411.5	609
Negative on discontinous edge	Short	263.5	390
	Long	229	338
positive	Short	287.5	425.5
	Long	141	208

edge		Coeficient	Momento
Negative on inside edges	Short	427	632
	Long	361	534
Negative on discontinous edge	Short	236	339
	Long	229	339
positive	Short	221	327
	long	132	195

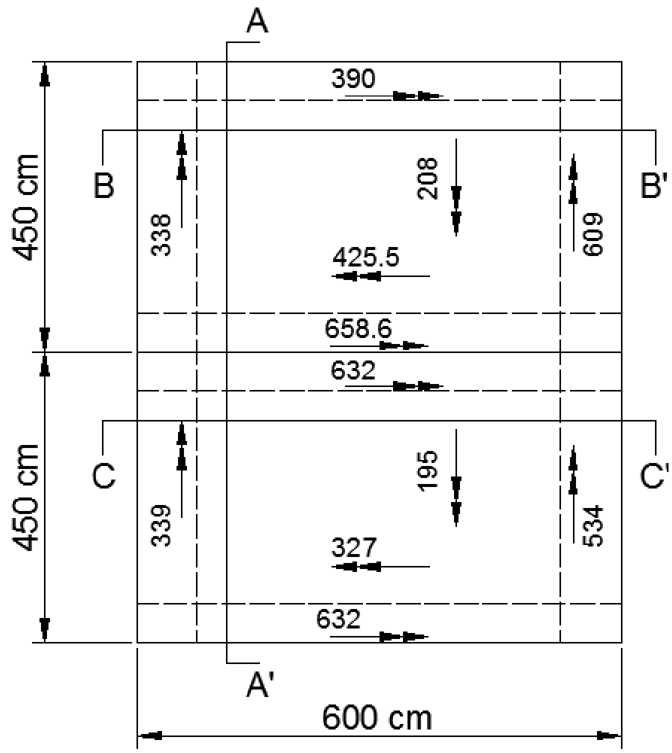


Figure 15.22

STEP 4

Distribution of moments

$$difference = 658.6 - 632$$

$$difference = 26.6$$

$$two\ thirds = 26.6\ (^2/_3)$$

$$two\ thirds = 17.7\ kg - m$$

Distributing half of the moment

$$\frac{17.7}{2} = 8.83\ kg - m$$

Final moments

$$658.6 - 8.85 = 649.75\ kg - m$$

$$632 - 8.85 = 640.85\ kg - m$$

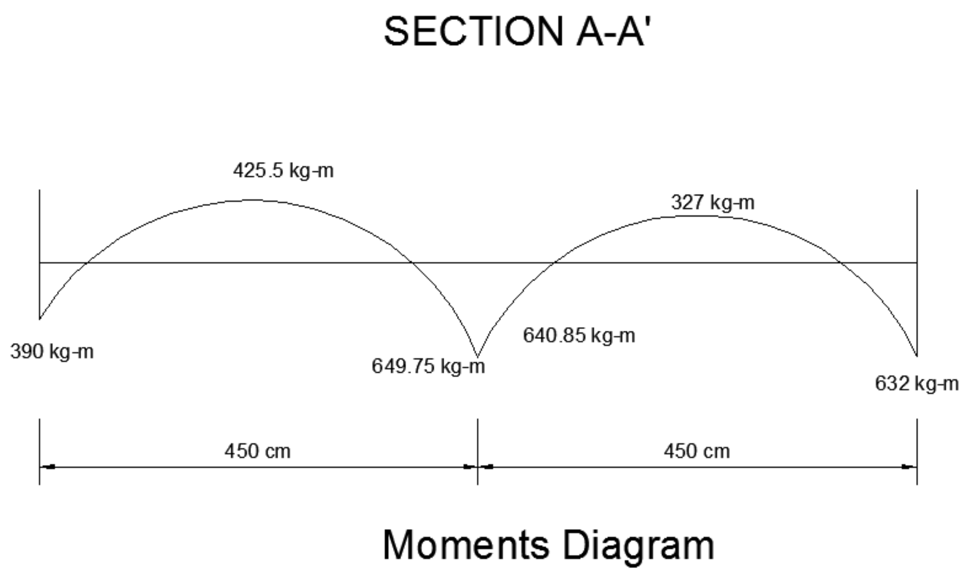


Figure 15.23

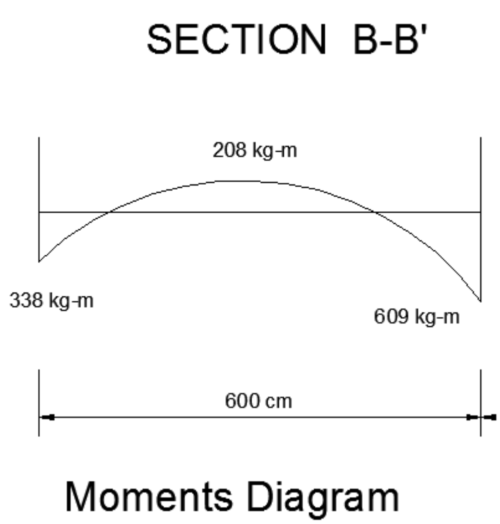


Figure 15.24

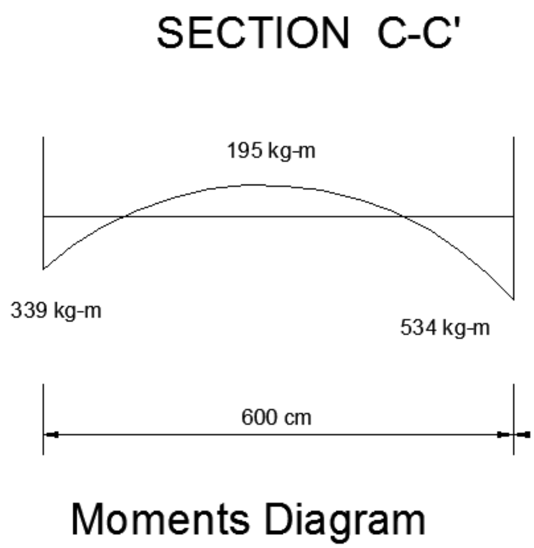


Figure 15.25

Section

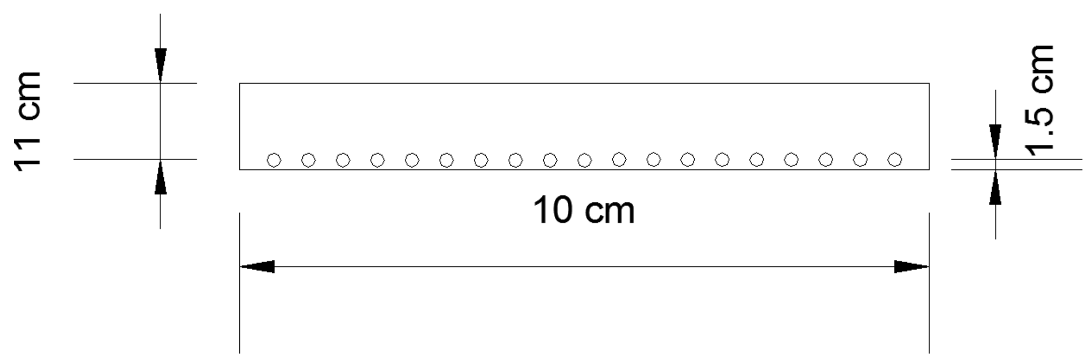


Figure 15.26

See Problem XVIII from the annex, page 239

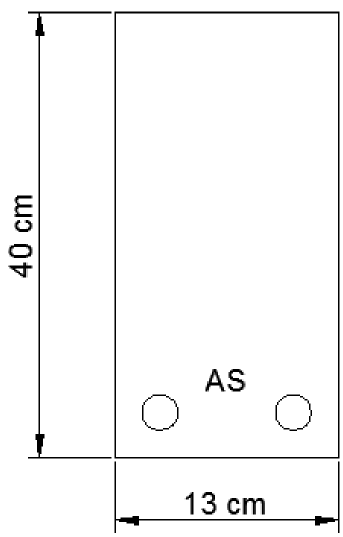
ANNEXES

PROBLEMS TO SOLVED

Problem I

Problem: Find the flexural strength of the beam.

Objective: show the student were the expressions for computing the resistance moment of a beam with only tension steel come from.



$f'c = 150$ kg/cm2

$fy = 4200$ kg/cm2

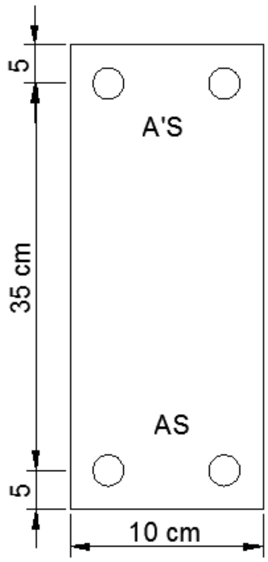
$As = 9.0$ cm2

Answer: $\boldsymbol{M_R = 10.70\ ton - m}$

Problem II

Problem: Find MR of the reinforced concrete section.

Objective: show the student how to calculate the strength with a trial and error procedure, an easiest way to determine the strength.



$f'c = 250 \text{ kg/cm2}$

$fy = 4200 \text{ kg/cm2}$

$As = 17 \text{ cm2}$

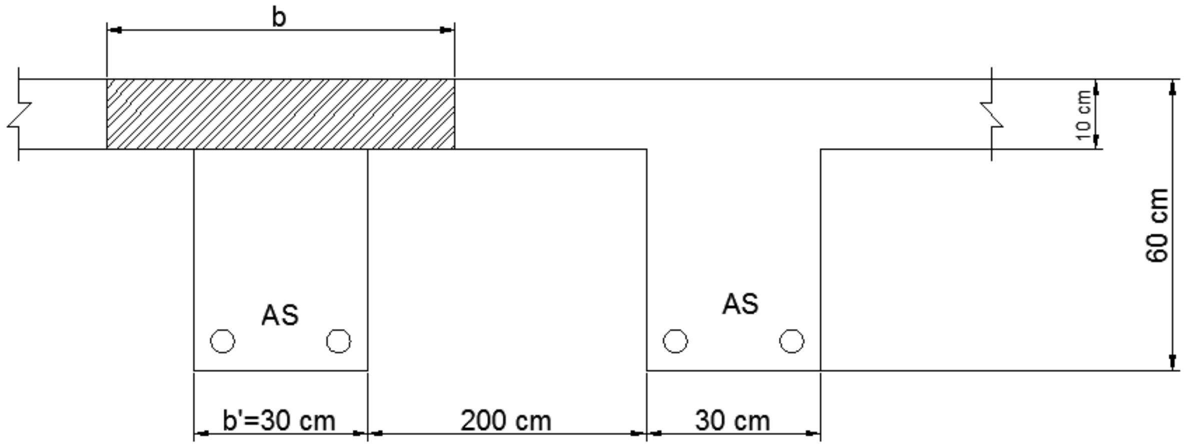
$A's = 4.0 \text{ cm2}$

For the correct Answer: ***T≈C***

Problem III

Problem: Find the flexural strength of the T-beam on the figure for two cases: As1= 9.0cm2 and As2= 17 cm2

Objective: To show the student how to compute the flexural strength of T- beam and compare its strength with two different quantities of steels.



$\mathrm{L} = 10\ \mathrm{m}$

$\mathrm{f'_c} = 250\ \mathrm{kg/cm2}$

$\mathrm{f_y} = 4{,}200\ \mathrm{kg/cm2}$

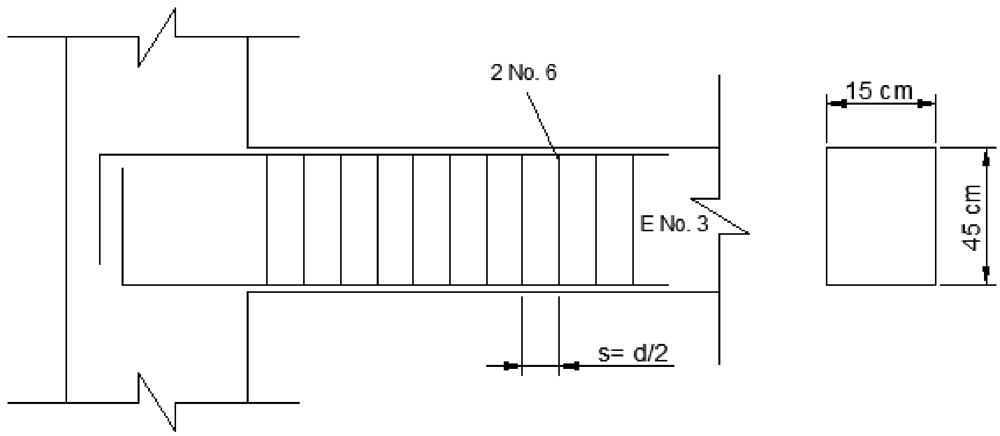
Answer: $\boldsymbol{M_n = 37.8\ ton - m}$

$\boldsymbol{M_R = 34\ ton - m}$

Problem IV

Problem: find the shear strength for the beam in the figure.

Objective: to show to the student how to compute the shear resistances of a beam.



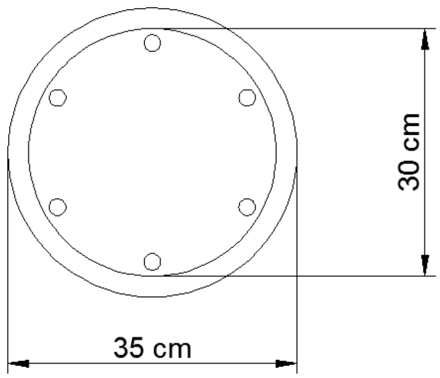
$$f'_c = 250 \text{ kg/cm2}$$

$$f_y = 4{,}200 \text{ kg/cm2}$$

Answer: $\boldsymbol{V_n = 14.3\ ton}$

Problem V

Problem: Compute first and second maximums for two spacing of spiral wire “s”
Objective: To compute the resistance of a column to axial forces.



$f'c = 2500\ kg/cm2$

$f'y = 4200\ kg/cm2$

$As = 6$ bars of No. 6

$S1 = 10\ cm$

$S2 = 15\ cm$

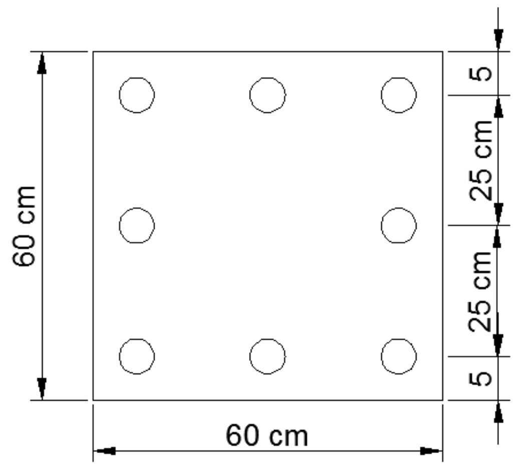
No. 2 bars of the espirals

Answer		***First maximum***	***Second maximum***
	S1	***187.96 ton***	***173.51 ton***
	S2	***187.96 ton***	***166.76***

Problem VI

Problem: construct the strength interaction diagram for the square column.

Objective: To compute the full range of eccentricities from cero to infinitive in order to construct a strength interaction diagram defining the failure load and the safe region and the failure region.



$f'c = 2500\ \text{kg/cm2}$

$f'y = 4200\ \text{kg/cm2}$

No. 8 bars

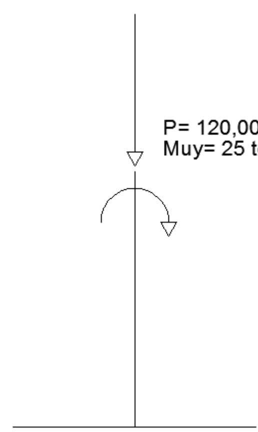
Answer		***Point 1***	***Point 2***	***Point 3***
	PRO	***780 ton***	***567.12 Ton***	***410.86 Ton***
	M	***0***	***47.34 Ton-m***	***64.48 Ton-m***

Problem VII

Problem: From a structural analysis of a structure the following forces are found to be acting on a column. Pu = 120,000 Kg and Muy = 25 ton-m

Propose a reinforced concrete section to resist the forces.

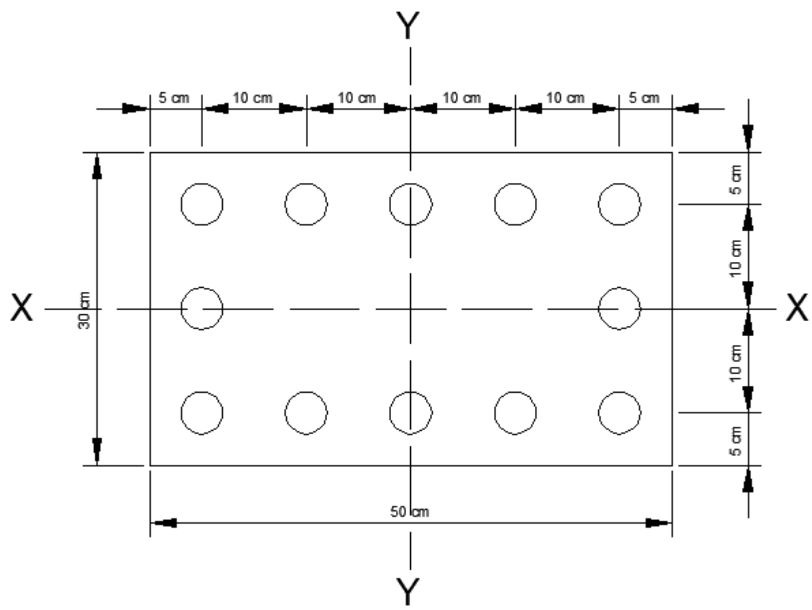
Objective: Propose a reinforcement concrete section to resist the forces that are acting in a column



Problem VIII

Problem: From a structural analysis of a structure, the following forces are found to be acting a column, Pu= 90,000 Kg, Muy= 13 ton-m and Mux= 10 ton-m. Check the adequacy of the trial design using the reciprocal method.

Objective: To propose a reinforcement concrete section to resist the axial force and the bending moments those are acting on a column.



$$\text{Pu} = 90{,}000\ \text{Kg}$$

$$\text{Muy} = 13\ \text{ton} - \text{m}$$

$$\text{Mux} = 10\ \text{ton} - \text{m}$$

$$\text{f}'_{c} = 250\ \text{kg/cm2}$$

$$\text{f}_{y} = 4200\ \text{kg/cm2}$$

$$\text{A}_{s} = 12\ \text{bars}\ \#\ 5$$

Answer	***Pn***	***Pr***
	35.34 Ton	***28.27 Ton***

Problem IX

Slender columns. Nonsway frame

Figure 1 shows and elevation view of a multistory concrete frame, with beams of 100 cm wide and 40 cm height. The clear height of columns is 600 cm. The interior columns are tentative dimensioned 35x35 cm. The frame is effectively braced against sway movement by stair and elevator shafts (not shown in the figure). The structure will be subjected to vertical dead and live loads. An analysis of first order indicated the following loads acting on the column C3:

Dead loads	Live loads
P = 90 ton	P = 87,000 Kg
M_2 = 0.500 ton-m	M_2 = 20 ton-m
M_1 = -0.500 ton-m	M_1 = 17 ton-m

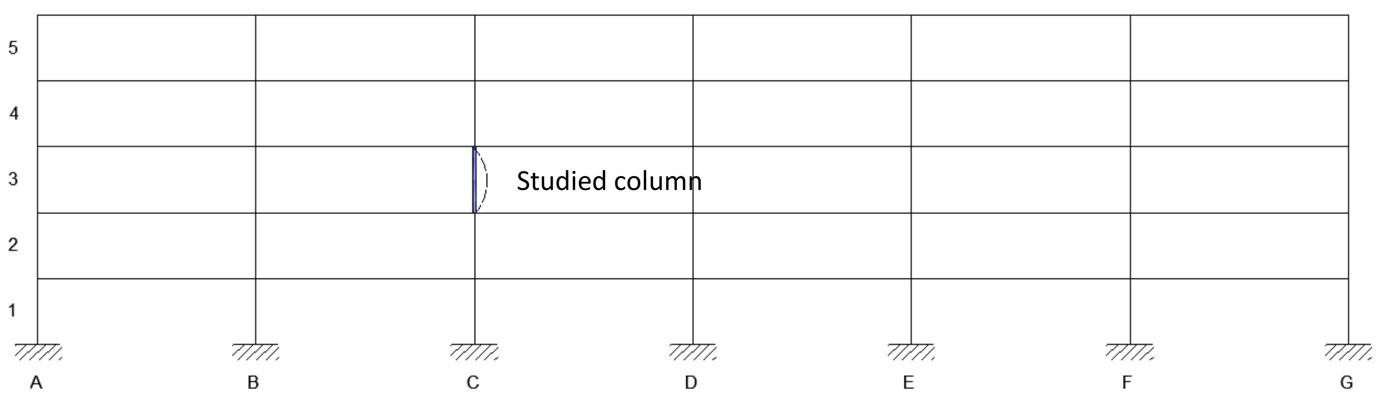
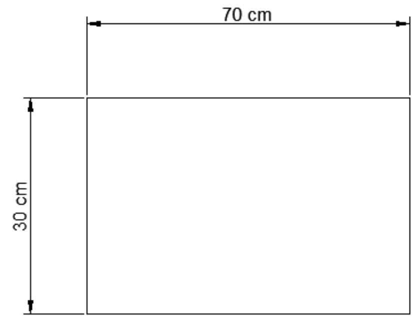


Figure 1. Elevation view

Problem X

Problem: Calculate the strength to torsion of the following concrete section.

Objetive: To show the student the effect of torsion on a concrete section.



$$f'c = 250 \text{ kg/cm2}$$

$$h = 30 \text{ cm}$$

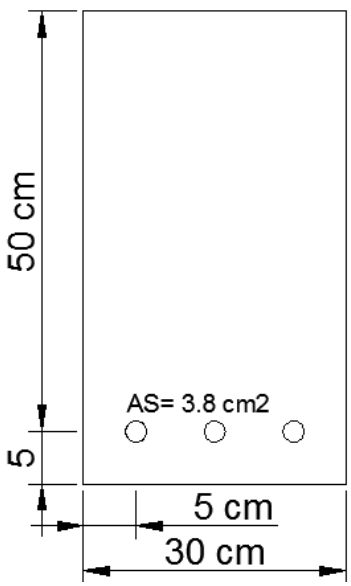
$$b = 70 \text{ cm}$$

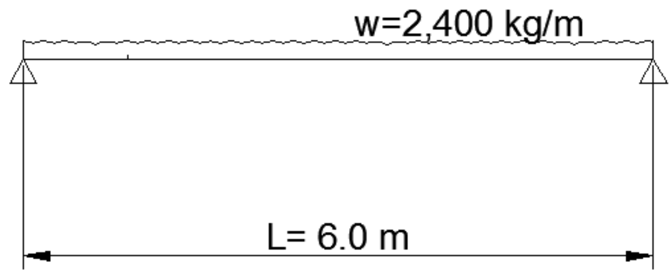
Answer $\boldsymbol{T_{cr} = 249.46.8\ ton/cm}$

Problem XI

Objetive: to show how to compute the probable crack width in a bean

Problem:





$$f'_c = 250\ \mathrm{kg/cm2}$$

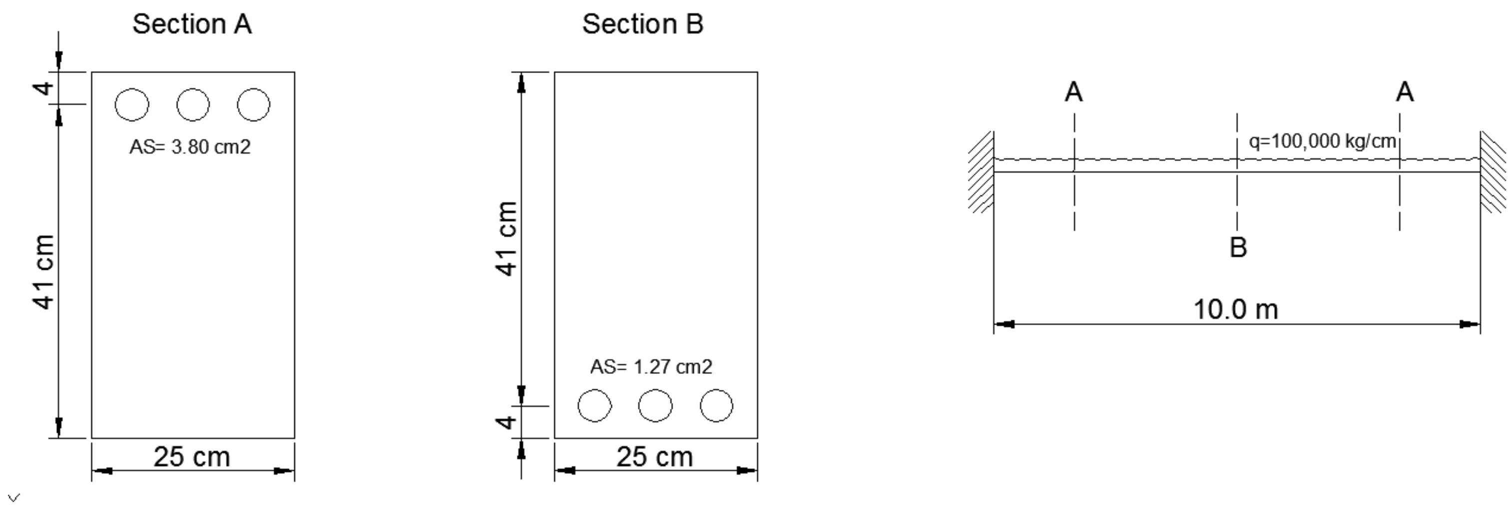
$$f_y = 4200\ \mathrm{kg/cm2}$$

Answer $\boldsymbol{w_{max} = 0.0559\ cm}$

Problem XII

Objective: to compute the Total deflection.

Problem:



$$f'_c = 250 \text{ kg/cm2}$$
$$f_y = 4200 \text{ kg/cm2}$$

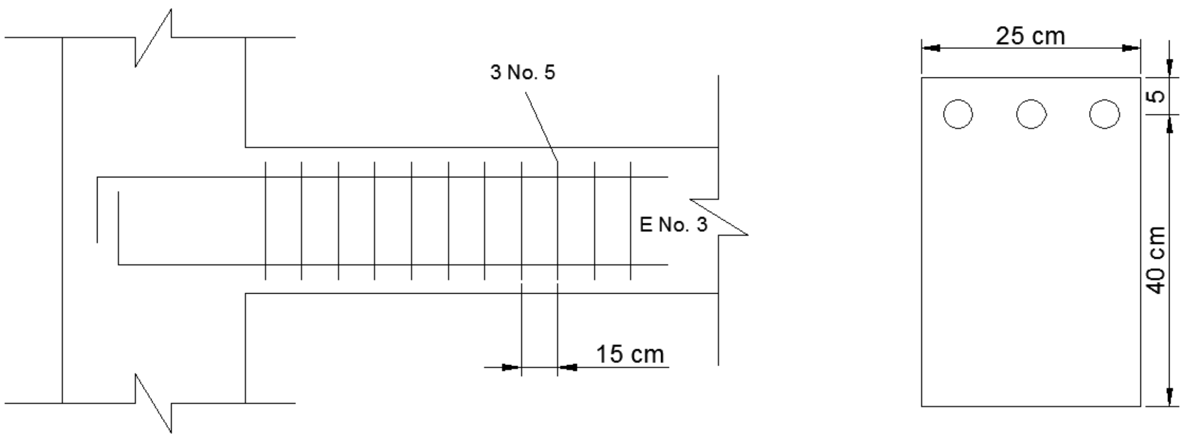
Answer: $\boldsymbol{inmediate\ \delta}$: ***3.28 cm***

$\boldsymbol{long\ term\ \delta}$: ***5.88 cm***

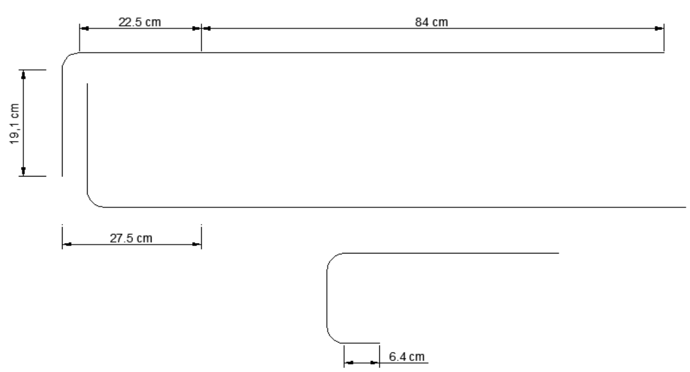
Problem XIII

Objetive: to compute the development length for non hooked and hooked bars

Problem: calculate the development length for the bar resisting the negative moment.



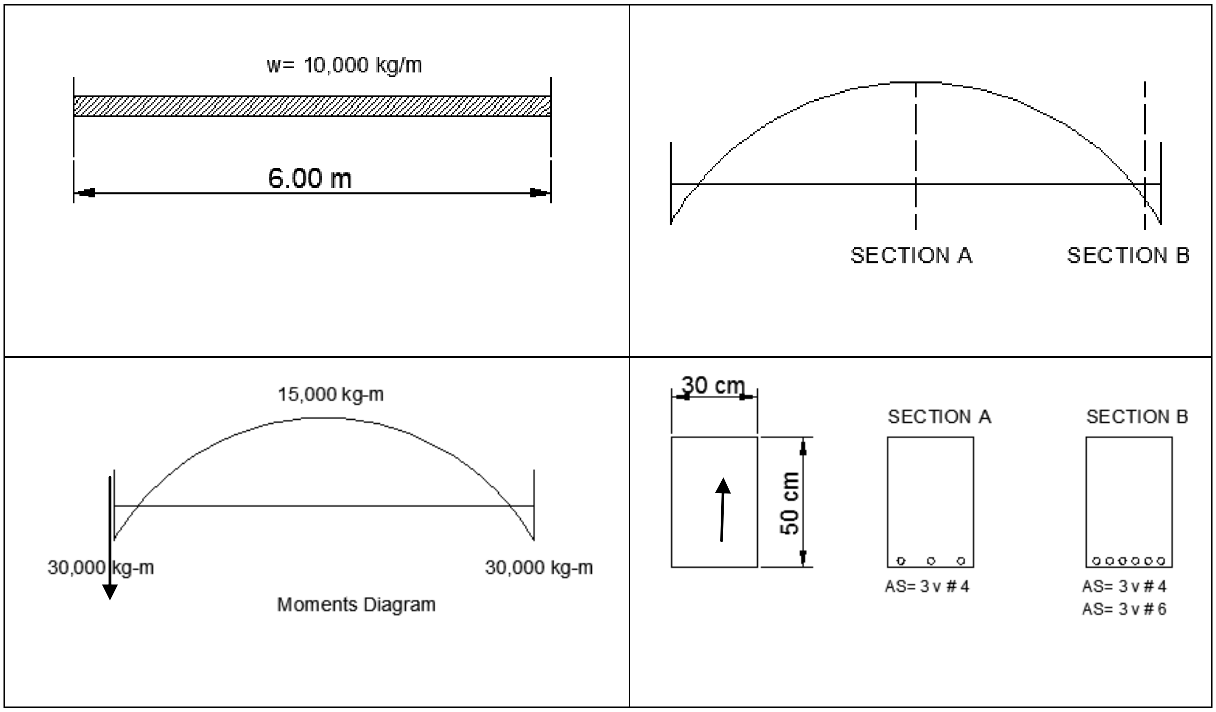
Answer:



Problem XIV

Objective: STEEL COUTING

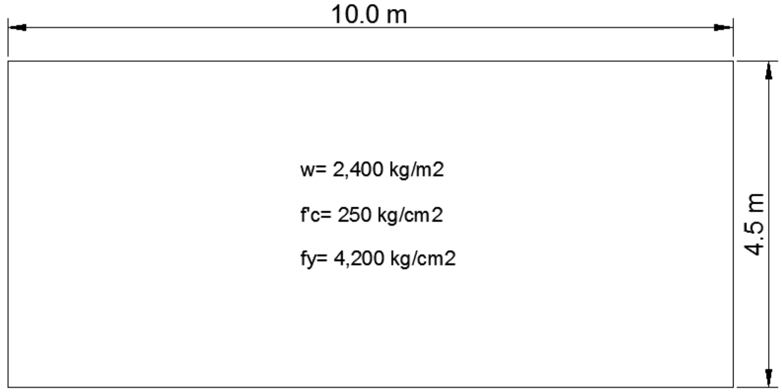
Problem:



Problem XV

Objective: Design the one-way slab being supported on its long edges. Consider that the slab was built integral with the support

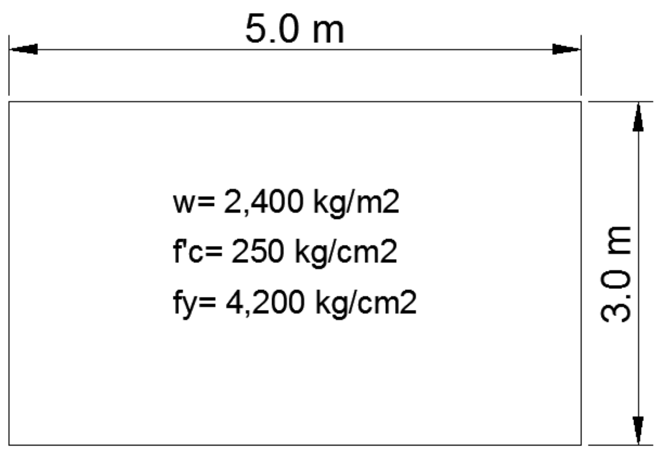
Problem:



Problem XVI

Objective: Design the two-way slabs being supported on its four edges.

Problem:



Problem XVII

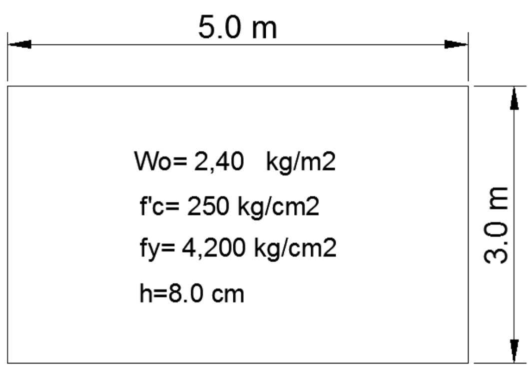
Objective: design a two-ways lab using the coefficients method (NTC-4004 page 142)

Problem: design the two-ways slab being supported on its four edges.

Consider the slab being built monolithically with it supports.

Consider the slab not being build monolithically whit it supports

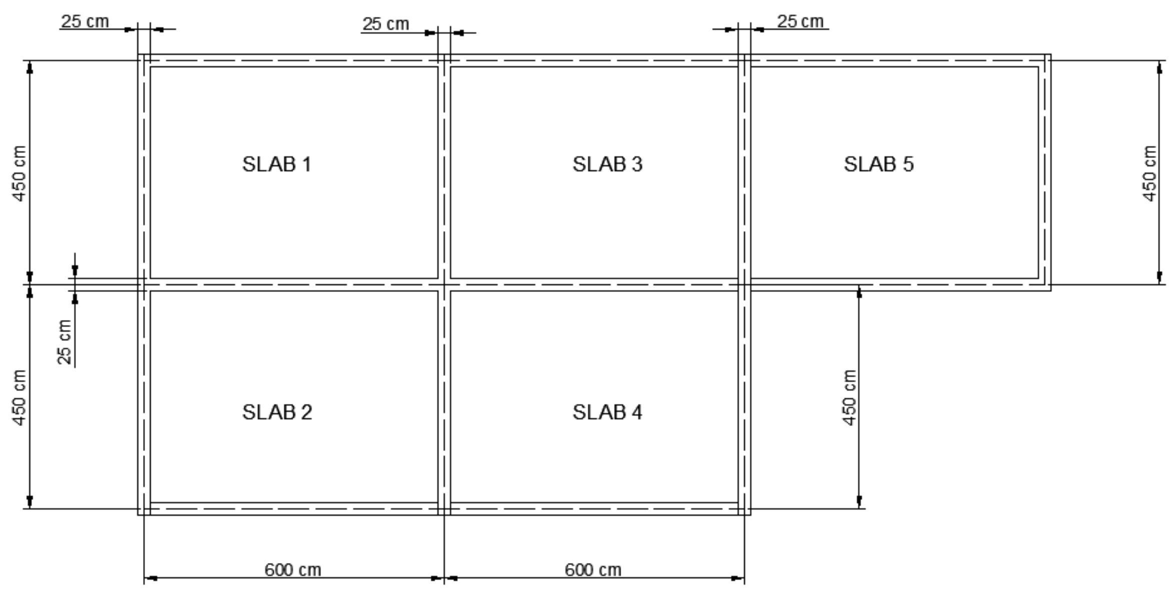
Note: apply the coefficients from the table 6.1 page 412



Problem XVIII

Objective: to determine the reinforcement for the slab. Consider the slab being cast monolithically with the supports.

Problem:



$$w(\text{live load}) = 190\ \text{kg/m2}$$
$$w(\text{cover}) = 250\ \text{kg/m2}$$
$$f'c = 250\ \text{kg/cm2}$$
$$fy = 4{,}200\ \text{kg/c}$$

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